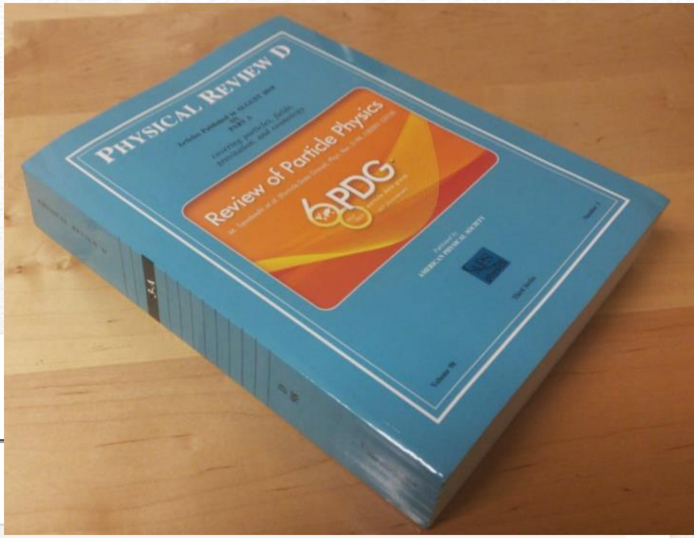
Analysis of Light Exotic Hadron measurements

Alessondro Pilloni

Snowmass, RF7 working group, September 30th, 2020

From data to the spectrum

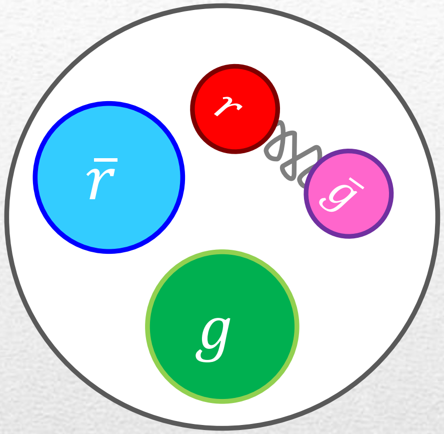


$a_1(1260)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	
250 to 600	**OUR ESTIMATE**				
$367 \pm 9 ^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	$190\ \pi^- \rightarrow \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		[1] AUBERT	2007AU	BABR	$10.6\ e^+ e^- \rightarrow \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	[2] LINK	2007A	FOCS	$D^0 \rightarrow \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[3] GOMEZ-DUMM	2004	RVUE	$\tau^+ \rightarrow \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\overline{p}\, p \rightarrow 2\pi^+ 2\pi^-$
460 ± 85	205	[4] DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	[5] ASNER	2000	CLE2	$10.6\ e^+ e^- \rightarrow \tau^+\tau^-,\ \tau^- \rightarrow \pi^-\pi^0\pi^0\nu_\tau$

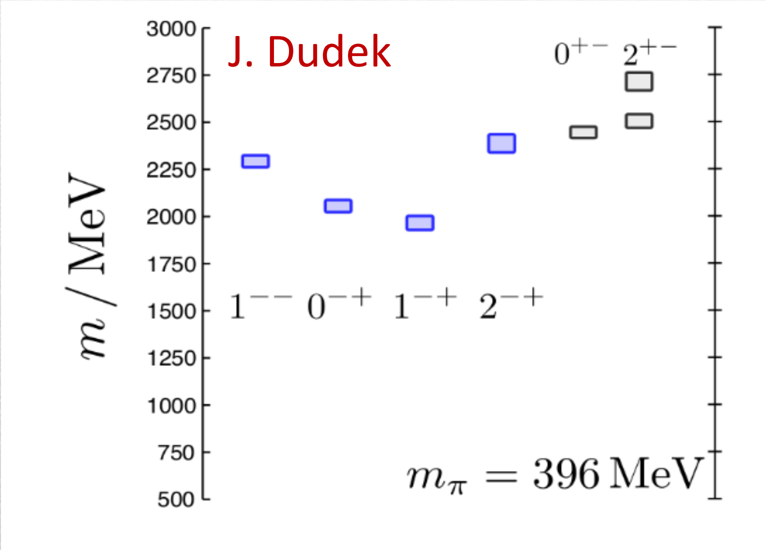
Theory support is mandatory to extract reliable physics information

Hybrid hunting



Gluonic quasiparticle
$J^{PC} = 1^{+-}$
mass $\sim$ 1.0–1.5 GeV

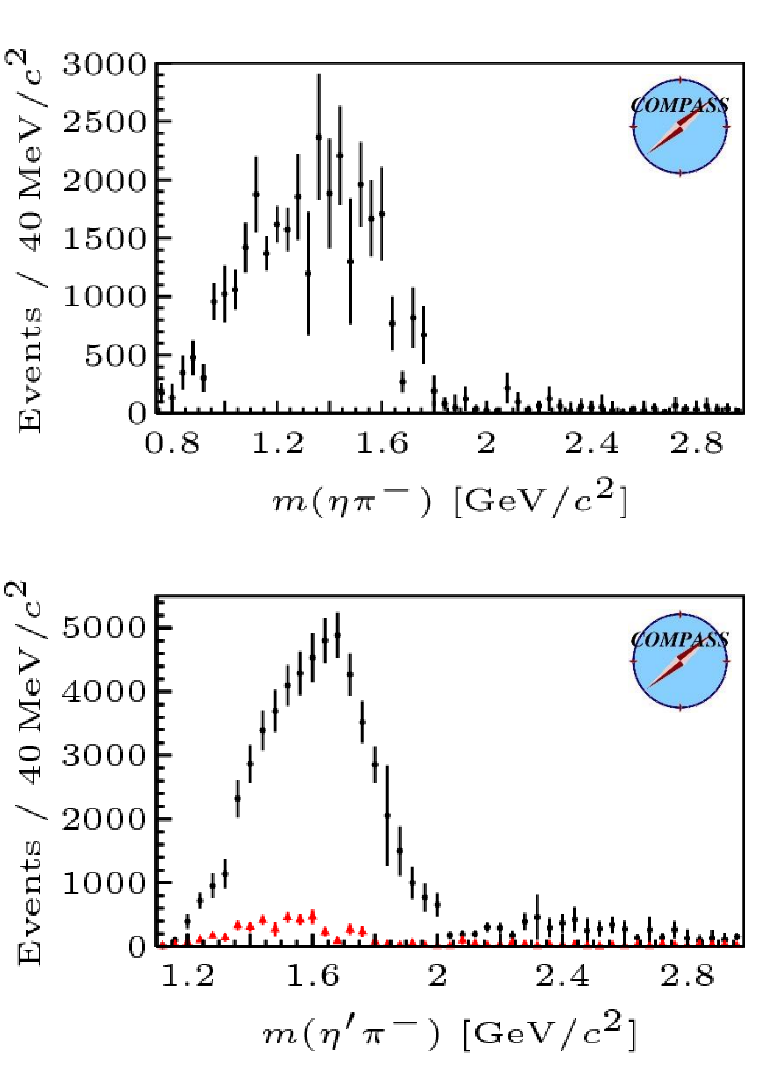
degenerate multiplet,
$J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$



Look for a π_1 state with $\boldsymbol{J^{PC} = 1^{-+}}$

decaying into $\begin{cases} \eta\,\pi \text{ and } \eta'\pi \\ \rho\,\pi \to 3\pi \\ b_1\pi \to 5\pi \end{cases}$

Two hybrid states???



$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level
Γ_1	$\eta\pi^0$	seen	
Γ_2	$\eta\pi^-$	seen	
Γ_3	$\eta'\pi$		

Neither lattice nor models predict two 1^{-+} states in this region!

$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

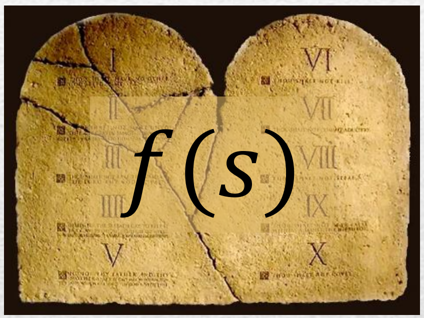
$\pi_1(1600)$ MASS	1662^{+8}_{-9} MeV
$\pi_1(1600)$ WIDTH	241 ± 40 MeV (S = 1.4)

Decay Modes

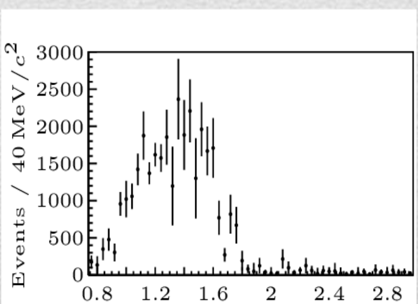
Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level
Γ_1	$\pi\pi\pi$	seen	
Γ_2	$\rho^0\pi^-$	seen	
Γ_3	$f_2(1270)\pi^-$	not seen	
Γ_4	$b_1(1235)\pi$	seen	
Γ_5	$\eta'(958)\pi^-$	seen	
Γ_6	$f_1(1285)\pi$	seen	

The flowchart(s)

1) You are given a model/theory



2) You calculate the amplitude

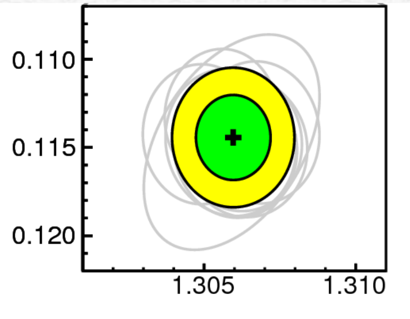


3) You compare with data.
Or you don't.

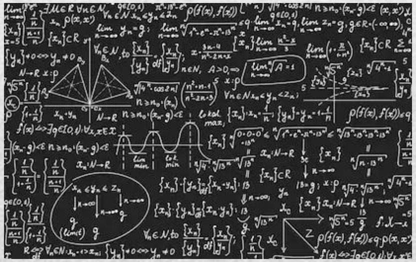
Predictive power ✓
Physical interpretation ✓
(within the model! ✘)
Biased by the input ✘

The flowchart(s)

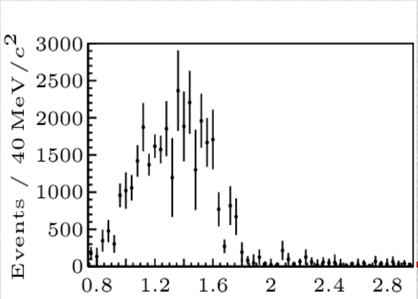
Less predictive power ✘
Some physical interpretation ✘
Minimally biased ✓



3) You extract physics

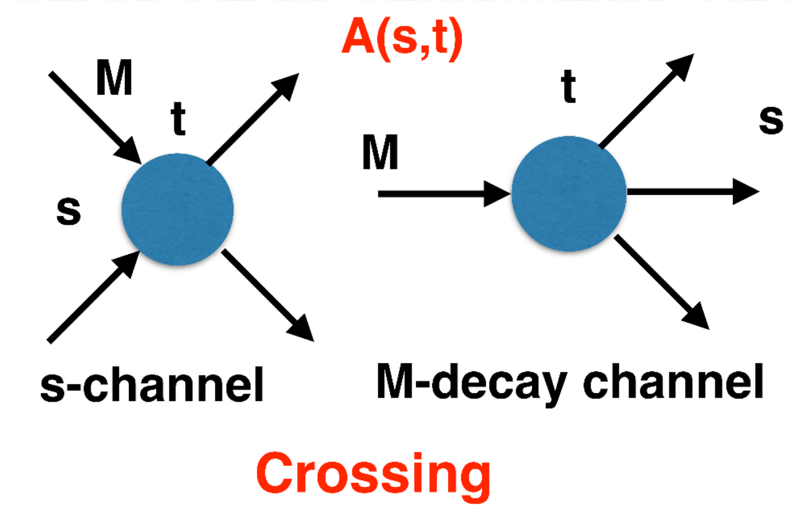


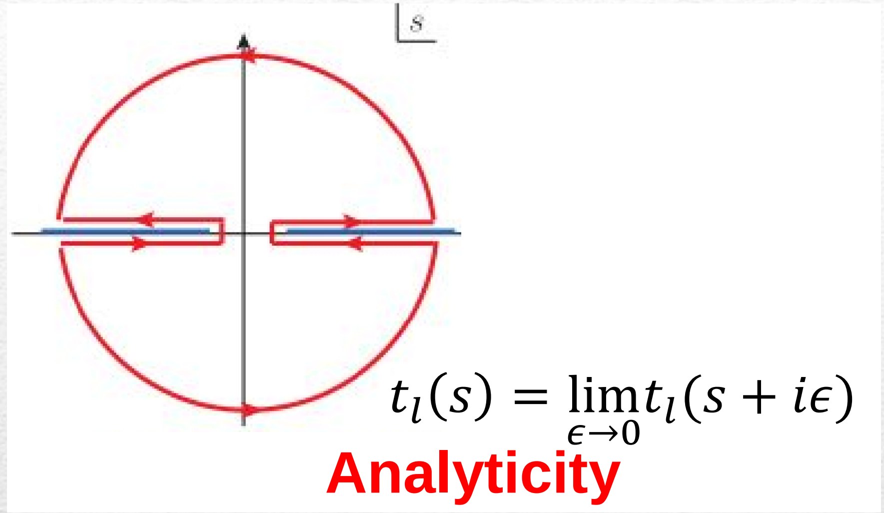
2) You choose a set of generic amplitudes

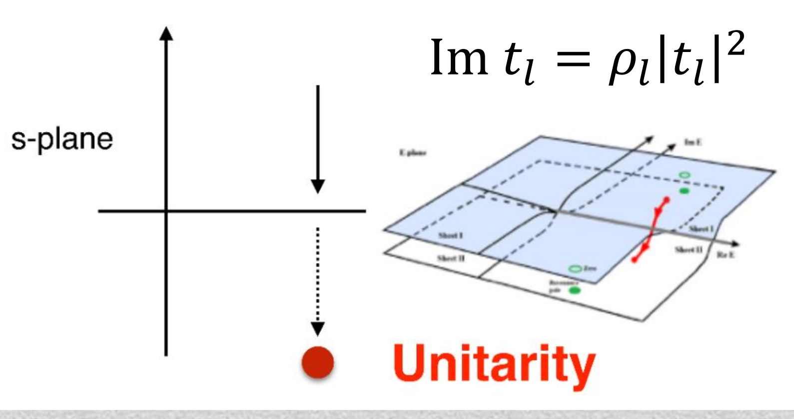


1) You start with data

S-Matrix principles







+ Lorentz, discrete & global symmetries

These are constraints the amplitudes have to satisfy, but do not fix the dynamics

They can be imposed with an increasing amount of rigor, to extract robust physics information

The «background» phenomena can be effectively parameterized in a controlled way

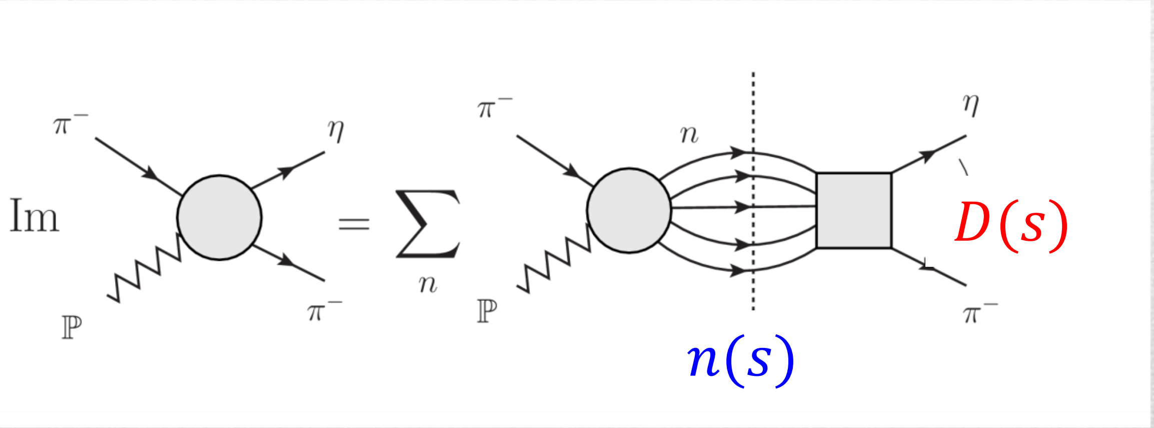
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the N/D method

Jackura, Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB

Rodas, AP *et al.* (JPAC), PRL

$$a(s) = \frac{n(s)}{D(s)}$$

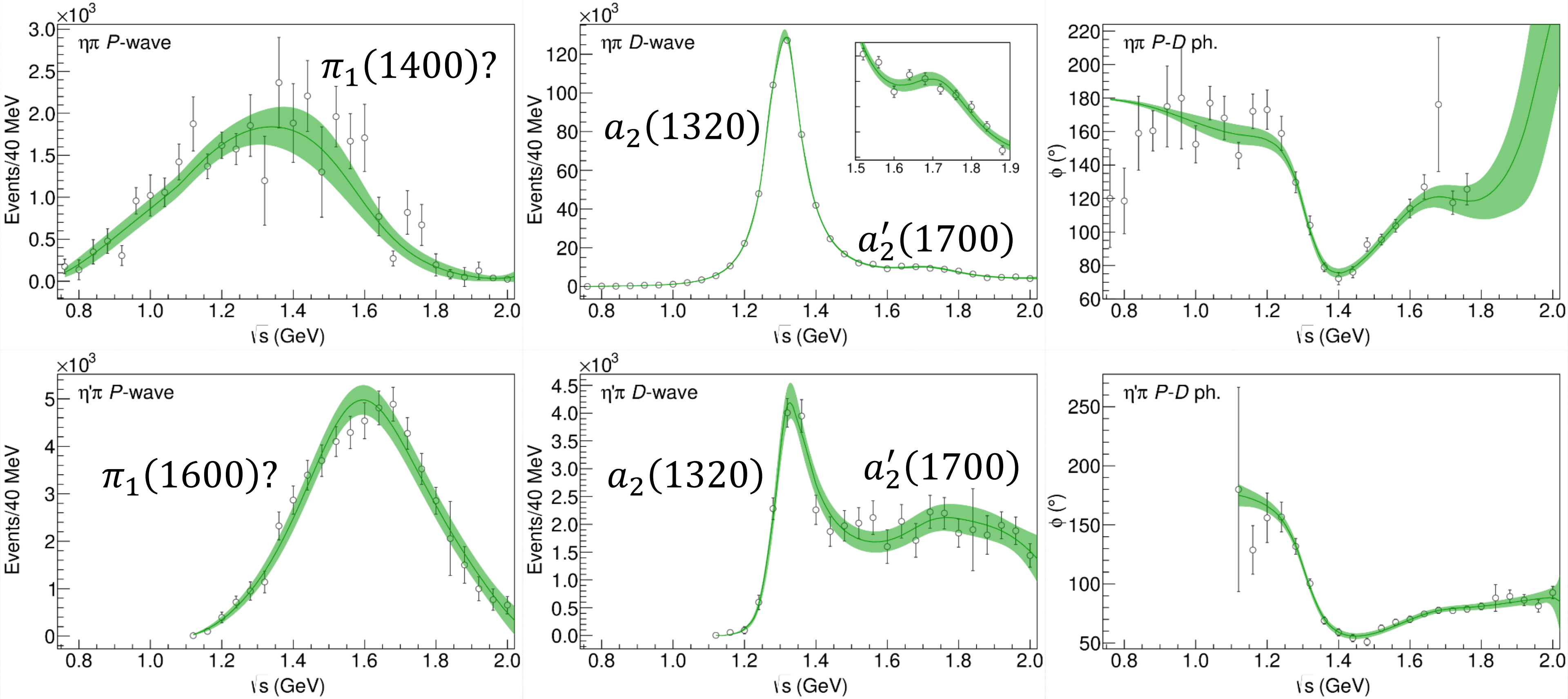


The $n(s)$ → background physics, process-dependent, smooth

The $D(s)$ contains all the Final State Interactions constrained by unitarity → universal

Fit to $\eta^{(\prime)}\pi$

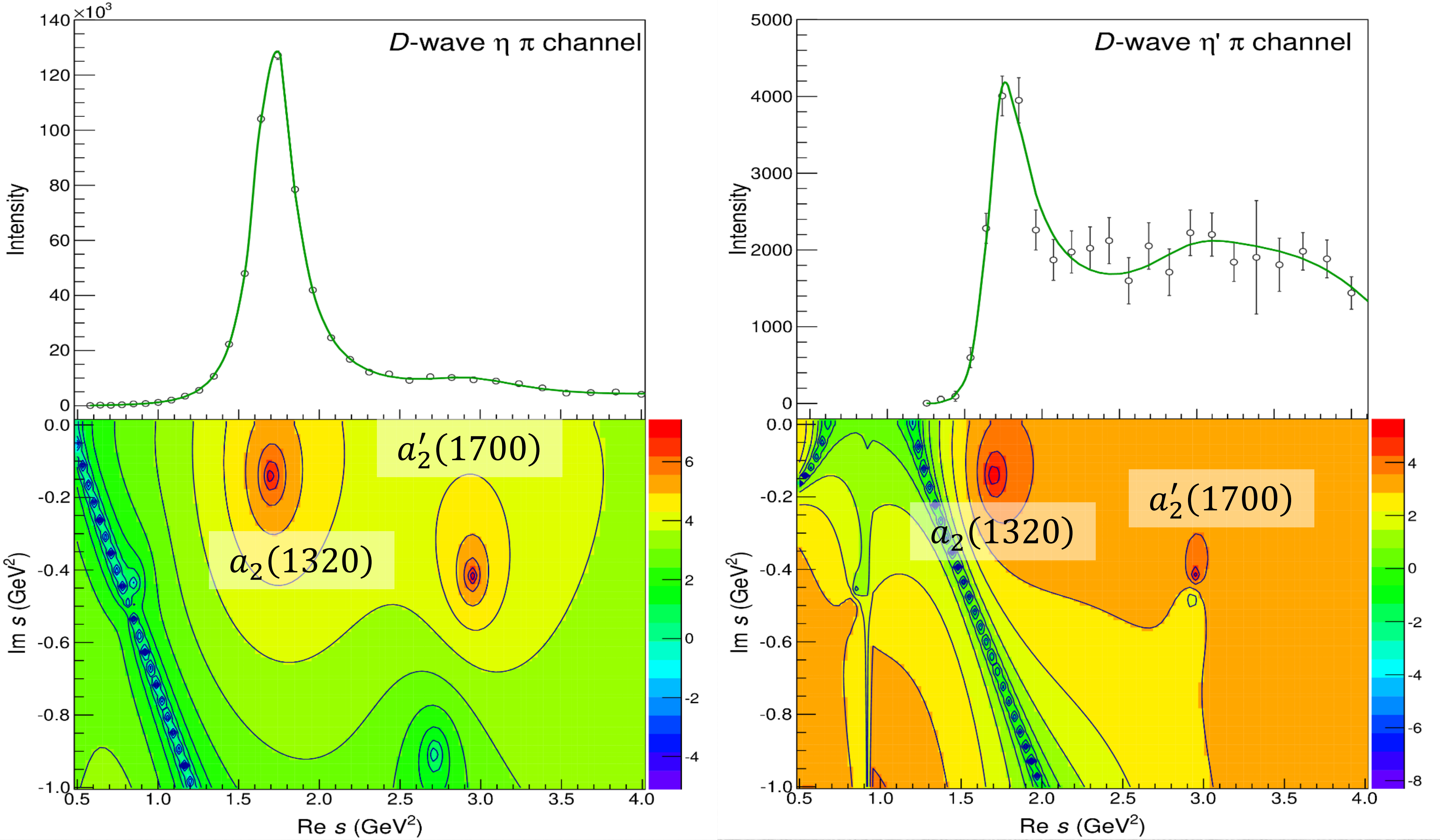
(data)



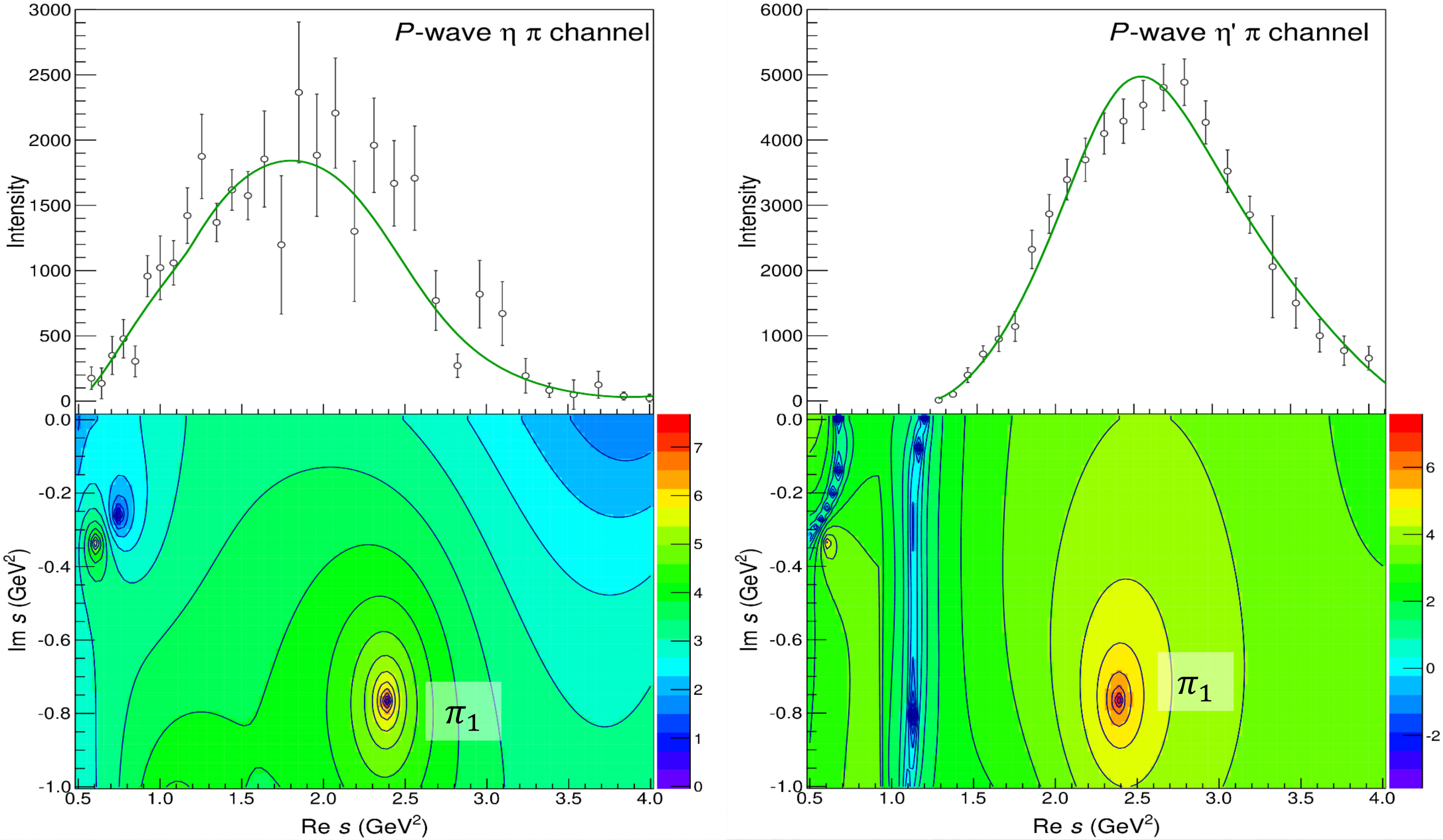
$J^{PC} = 1^{-+}$

$J^{PC} = 2^{++}$

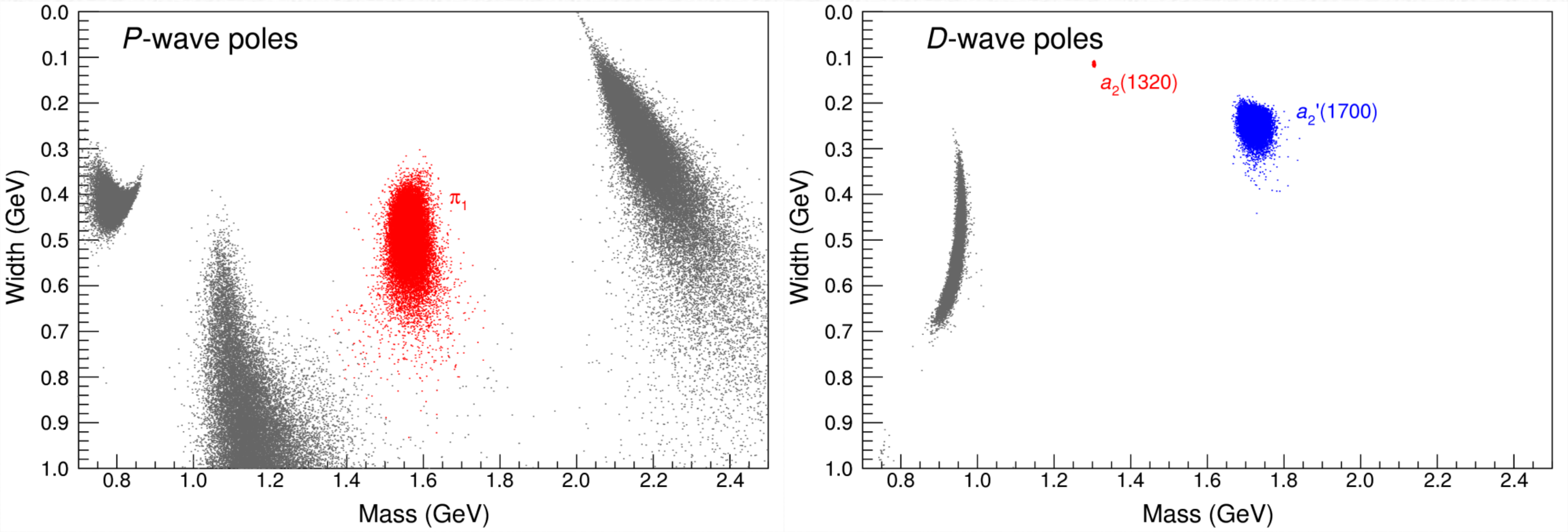
Pole hunting



Pole hunting



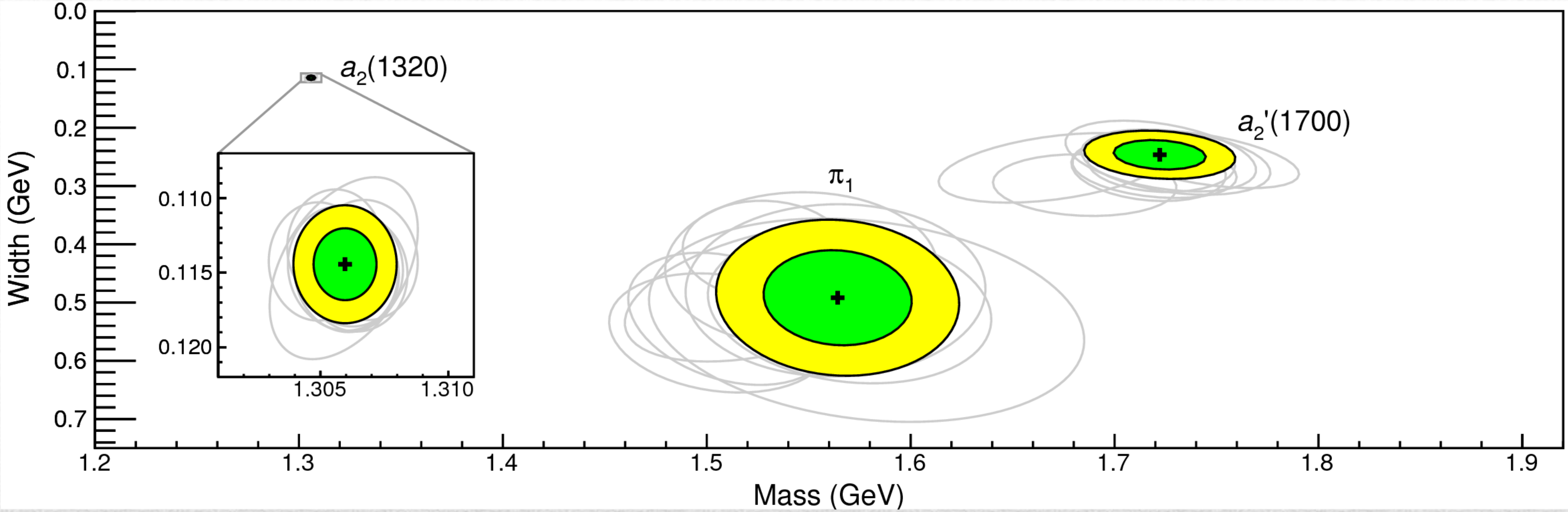
Statistical Bootstrap



We can identify the poles in the region $m \in [1.2, 2]$ GeV, $\Gamma \in [0, 1]$ GeV

Two clusters in *D*-wave: $a_2(1320)$ and $a_2'(1700)$
Only one stable cluster in P-wave: a single π_1

Final results

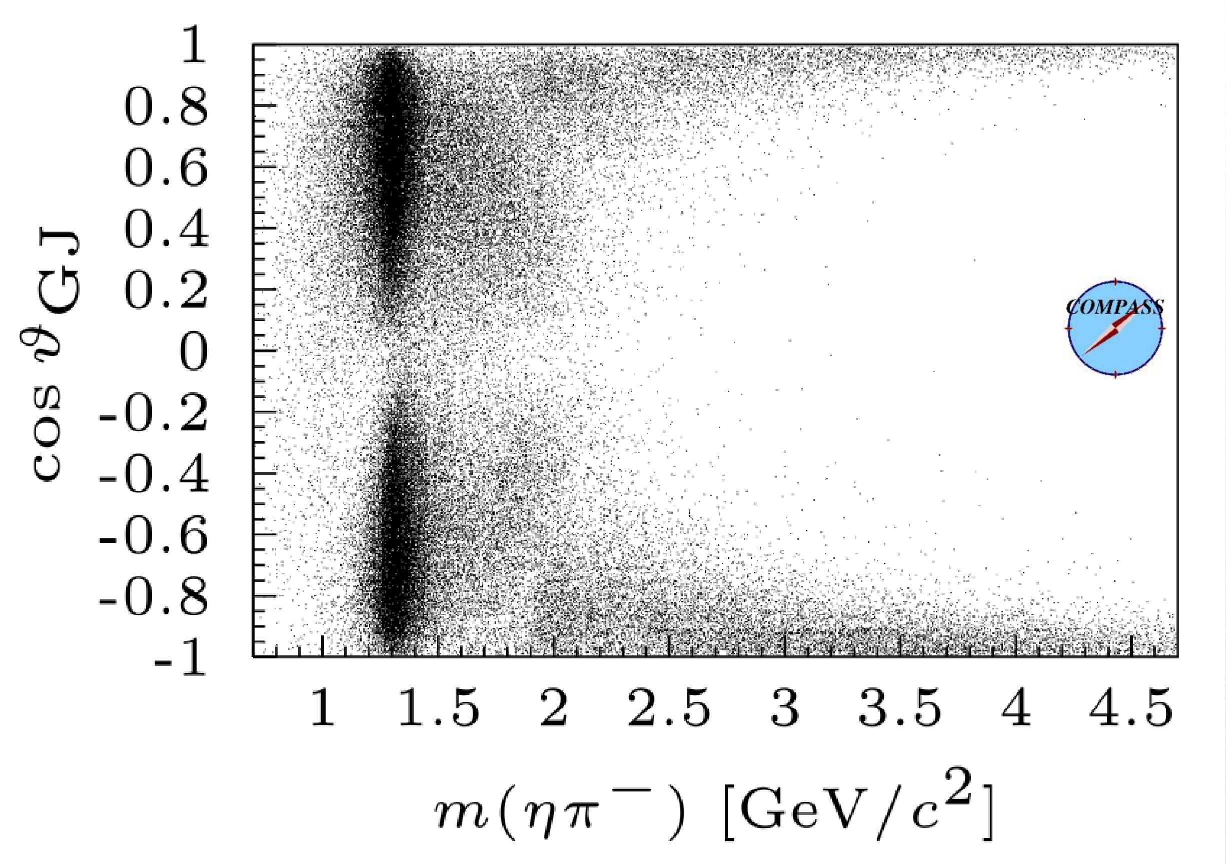


Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

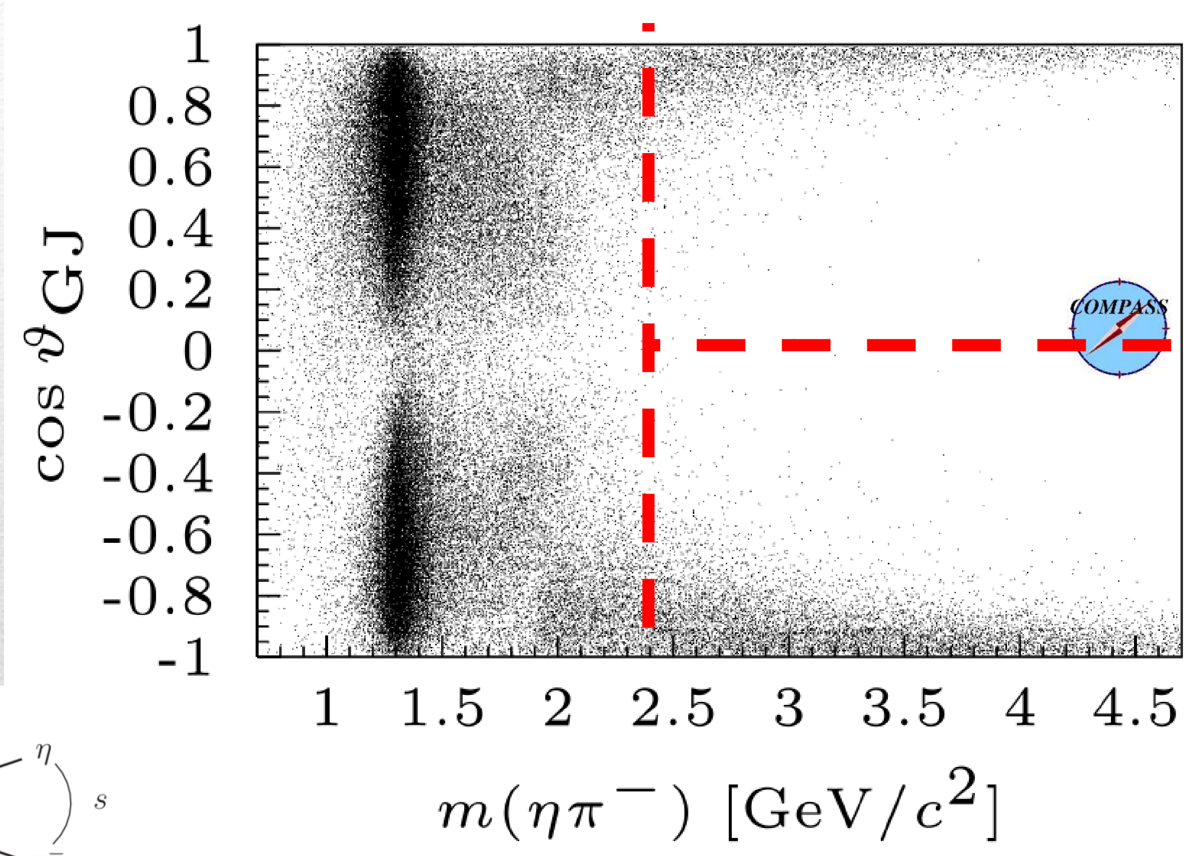
Agreement with theory is restored, cfr Jo's talk and 2009.10034

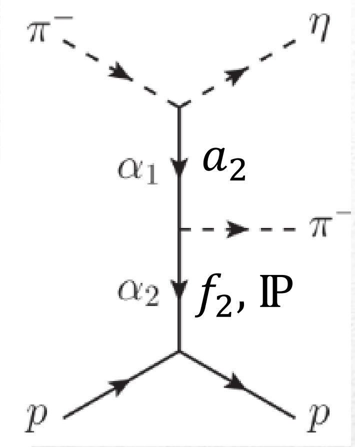
That's the most rigorous extraction of an exotic meson available so far!

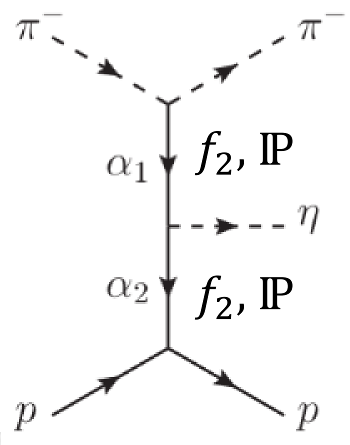
A new look to $\eta\pi$

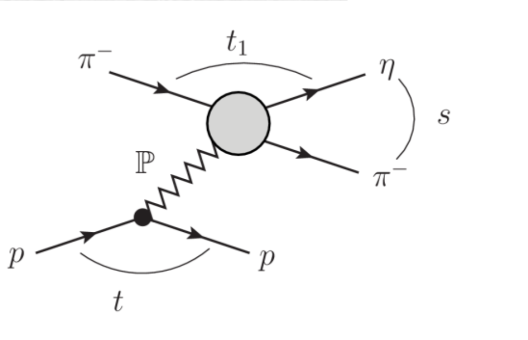


High energy





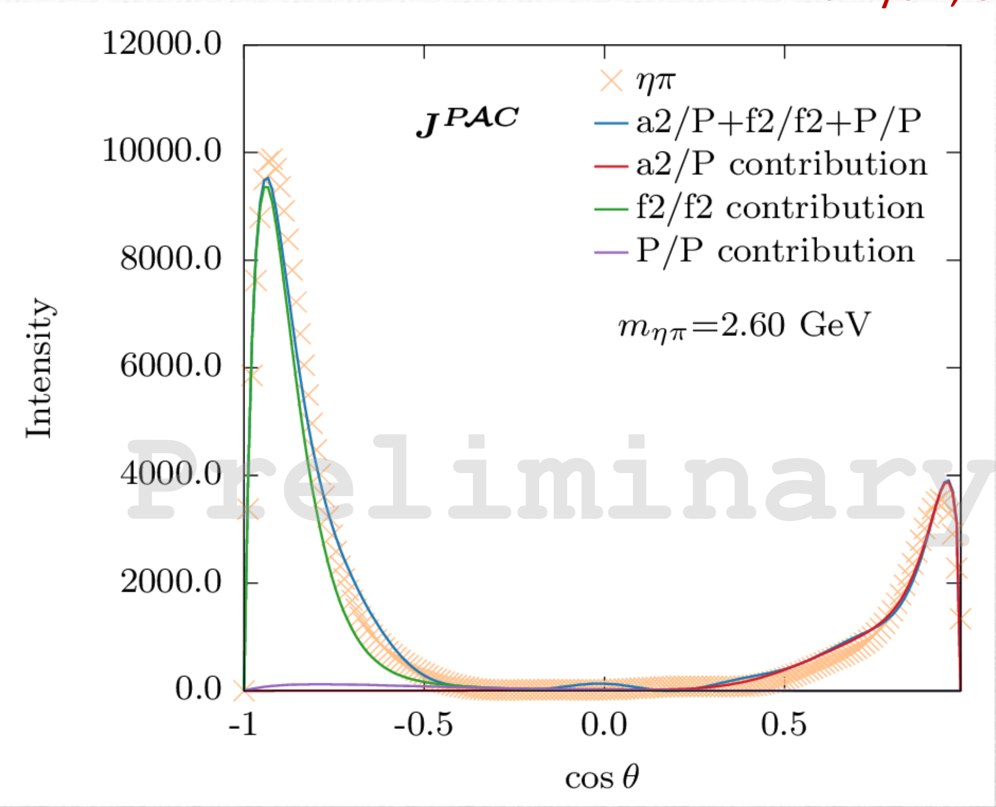




Three regions analytically connected (FESR)
Need to understand high energy first

Fit to high energy

Bibrzycki, *et al.* (JPAC), in progress

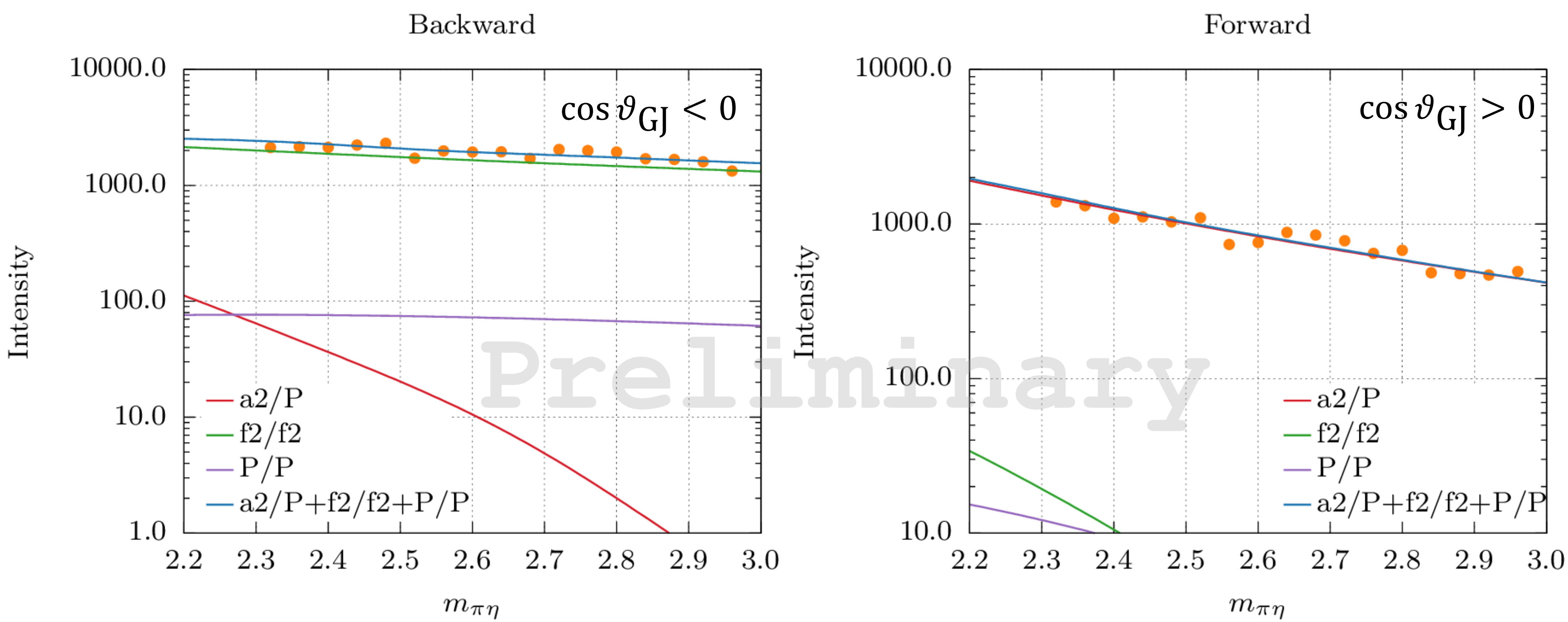


Double Regge Exchange Model Shimada *et al.*, NPB

Couplings of each diagram fitted to sum of partial waves in high energy region

Fit to high energy

Bibrzycki, *et al.* (JPAC), in progress



Double Regge Exchange Model Shimada *et al.*, NPB

Couplings of each diagram fitted to sum of partial waves in high energy region

Conclusions

Bottom-up approaches are important!

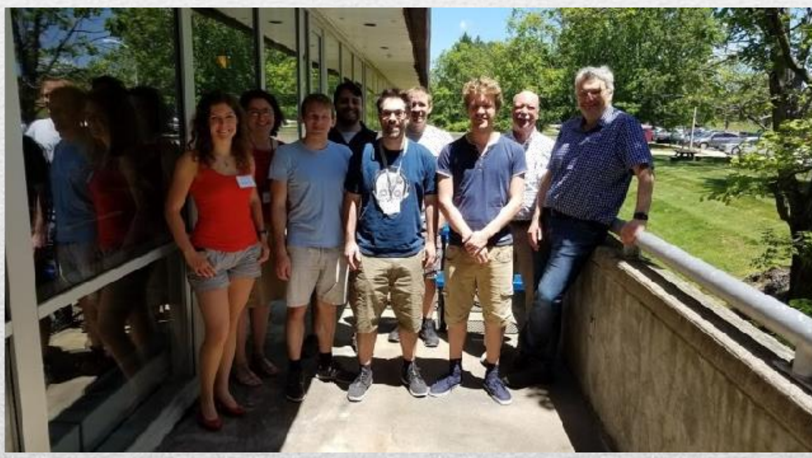
- They allow us to get the most out of high statistics data!
- Dispersive methods can improve the rigour and robustness in the extraction of the spectrum
- There is life beyond partial waves!
  High energy + analyticity help constrain the resonance region

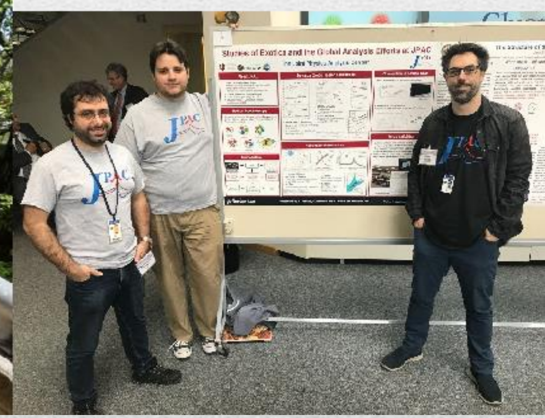
Thank you!

Joint Physics Analysis Center

- Joint effort between theorists and experimentalists in support of experimental data from JLab12 and other accelerator laboratories
- Cooperation between JPAC and experiments: co-authoring papers

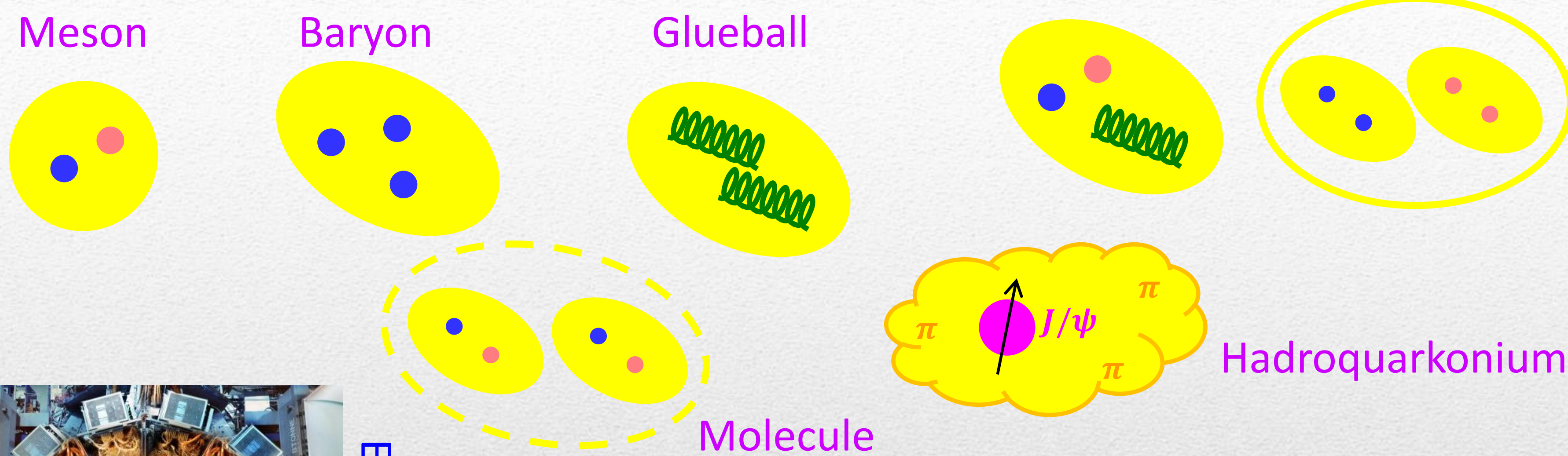
https://ceem.indiana.edu/jpac

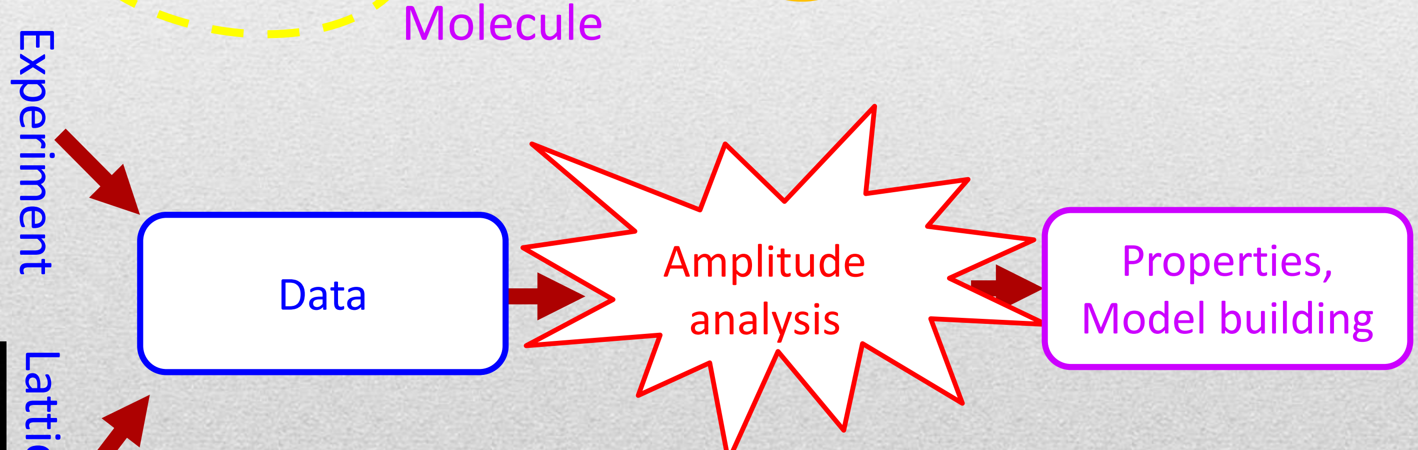




BACKUP

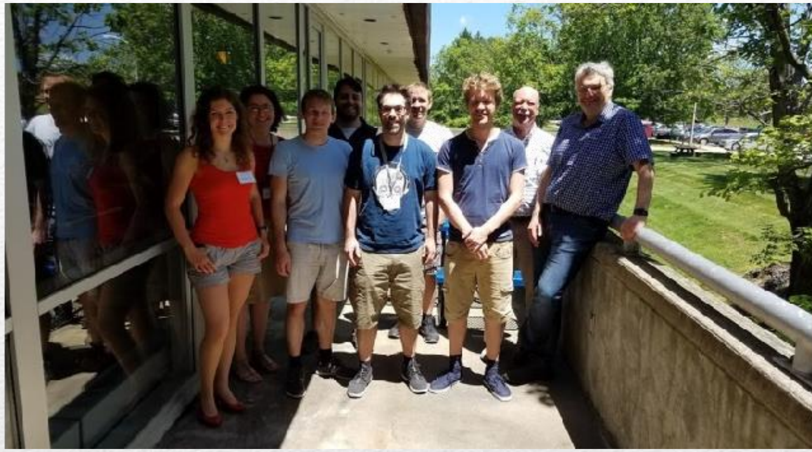
Building Hadrons

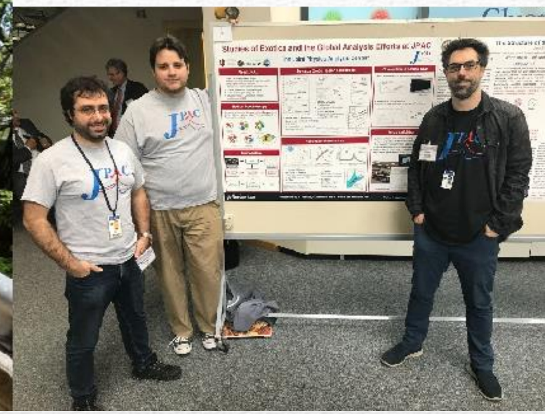




Interpretations on the spectrum leads to understanding fundamental laws of nature

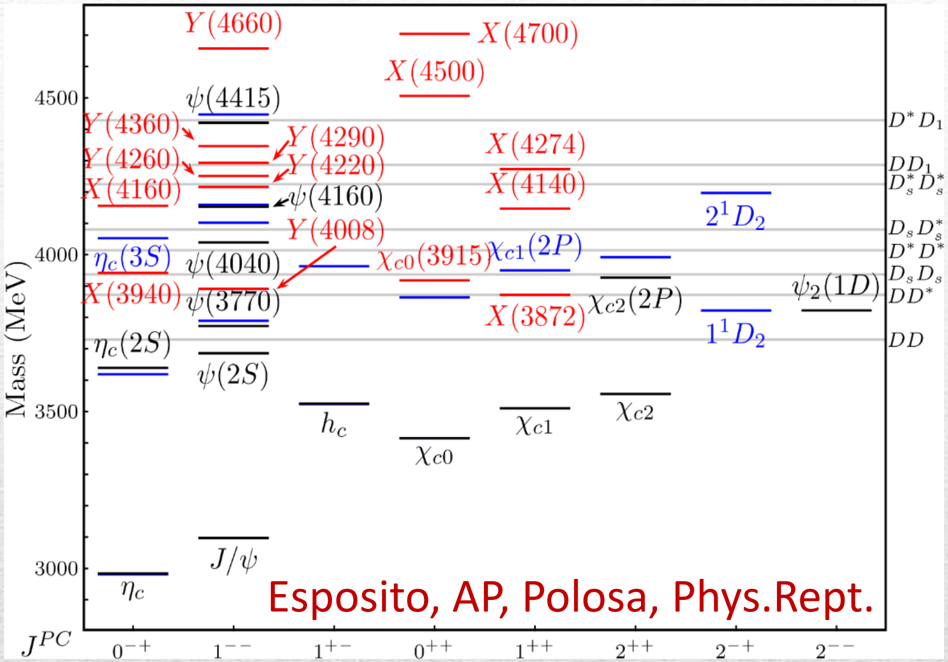
Joint Physics Analysis Center

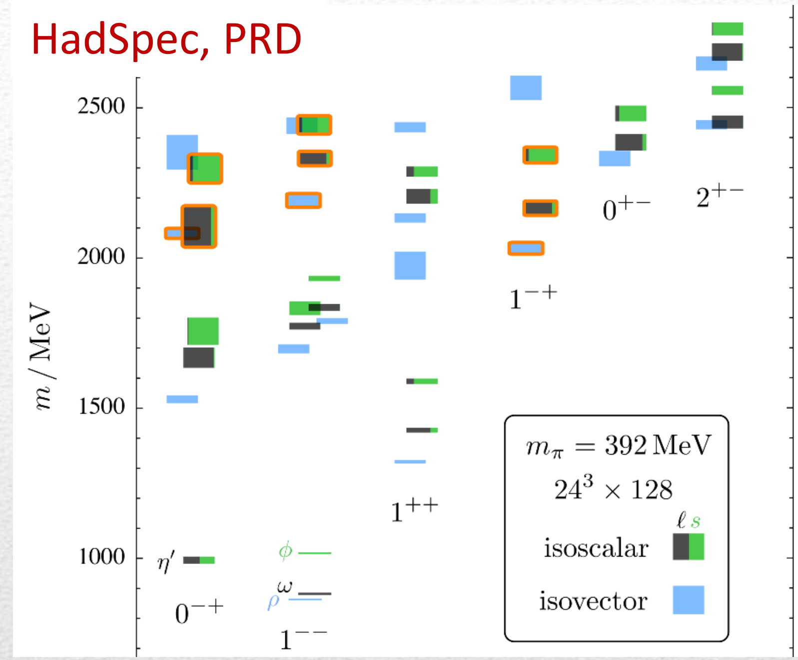


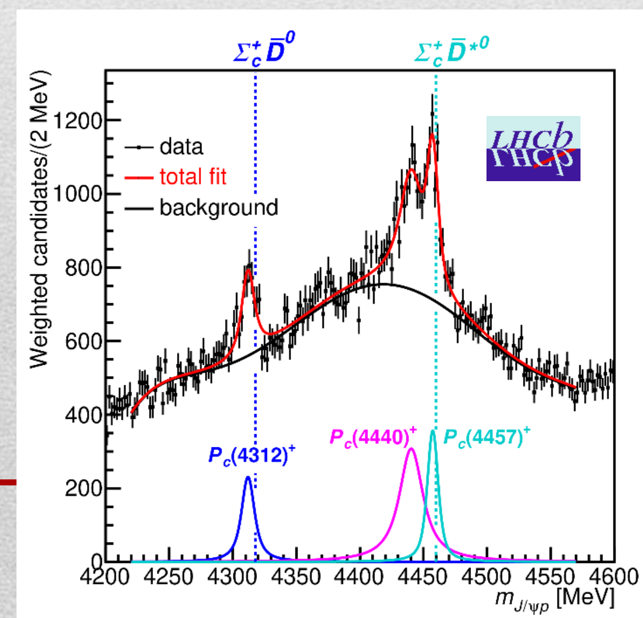


JPAC

QCD spectrum





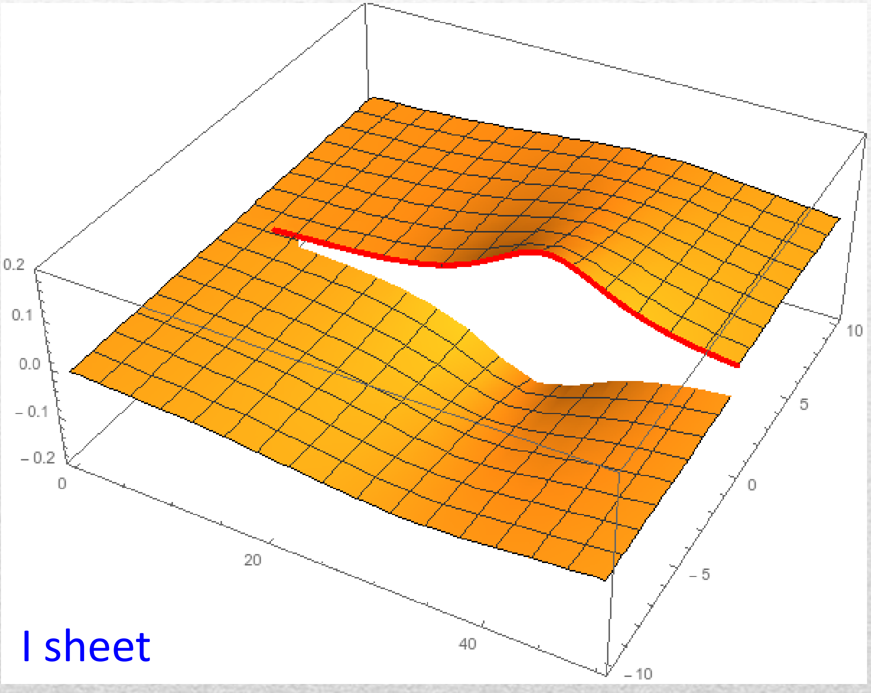


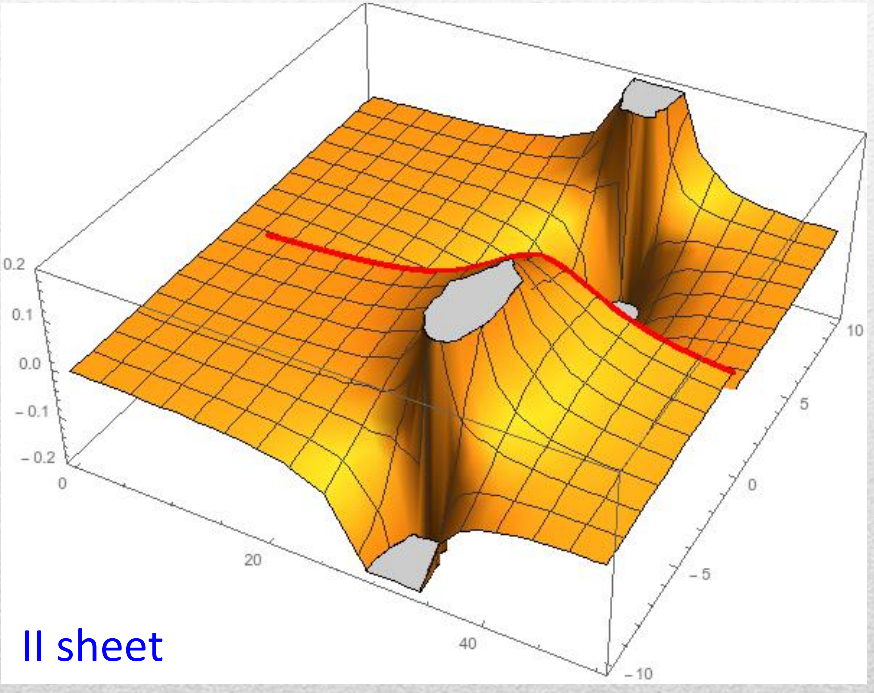
The spectrum of hadron excitations is incredibly rich

Several new observations beyond the minimal quark content

Unitarity & Pole hunting

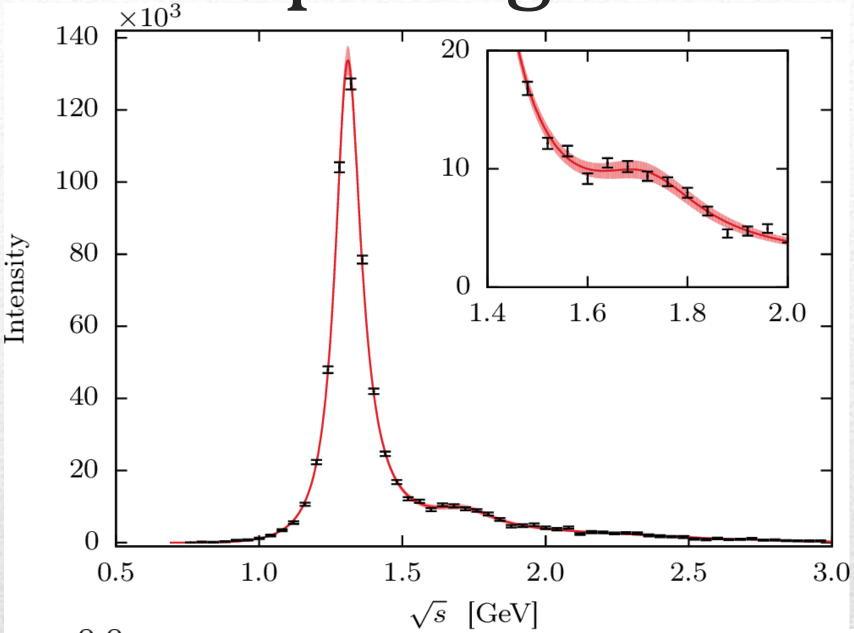
Unitarity creates a branch cut on the real axis,
two sheets continuosly connected





Finding resonances means writing analytic amplitude,
and hunting for poles in the complex plane

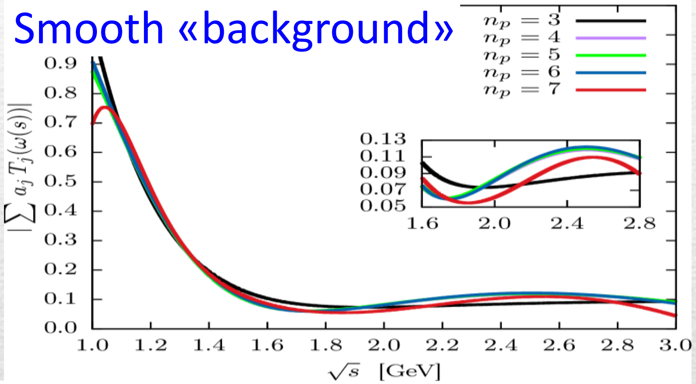
Recap: single channel $\eta\pi$

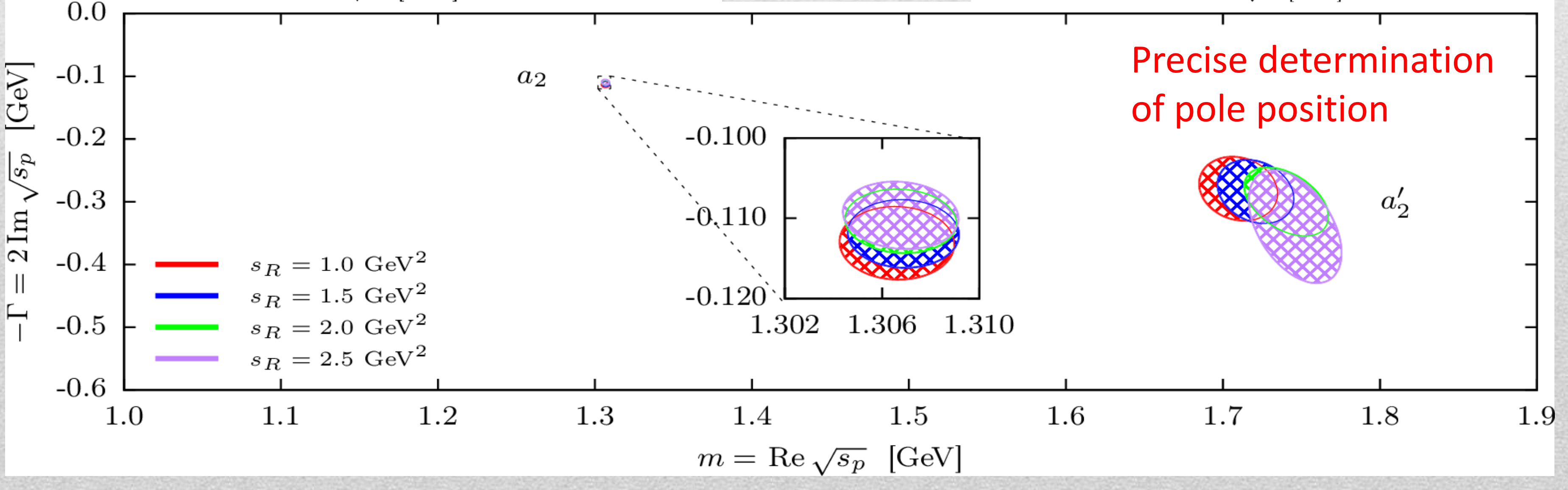


Test against the D-wave $\eta\pi$ data, where the a_2 and the a_2' show up

A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

Smooth «background»

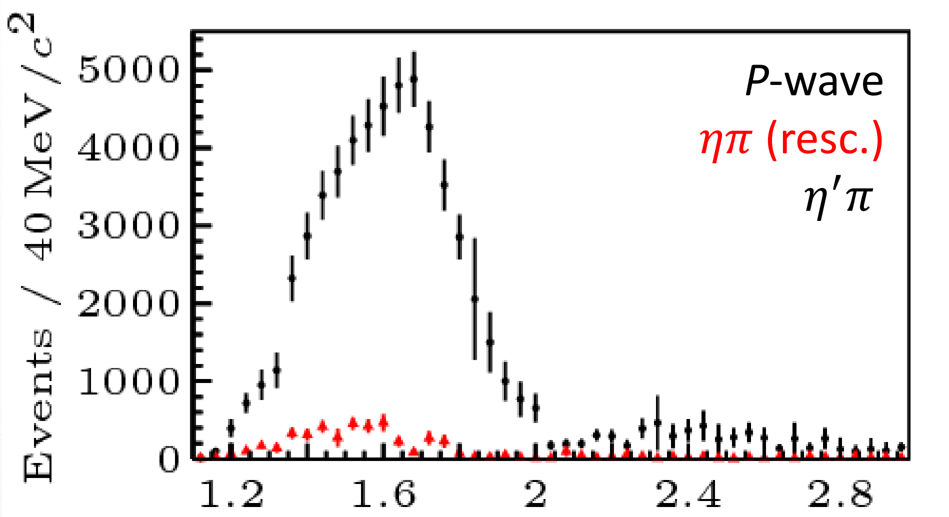


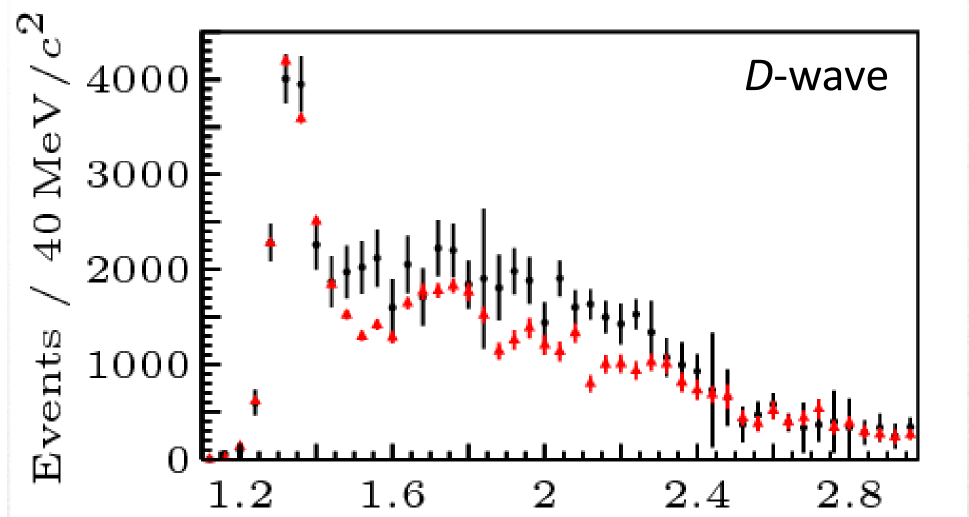


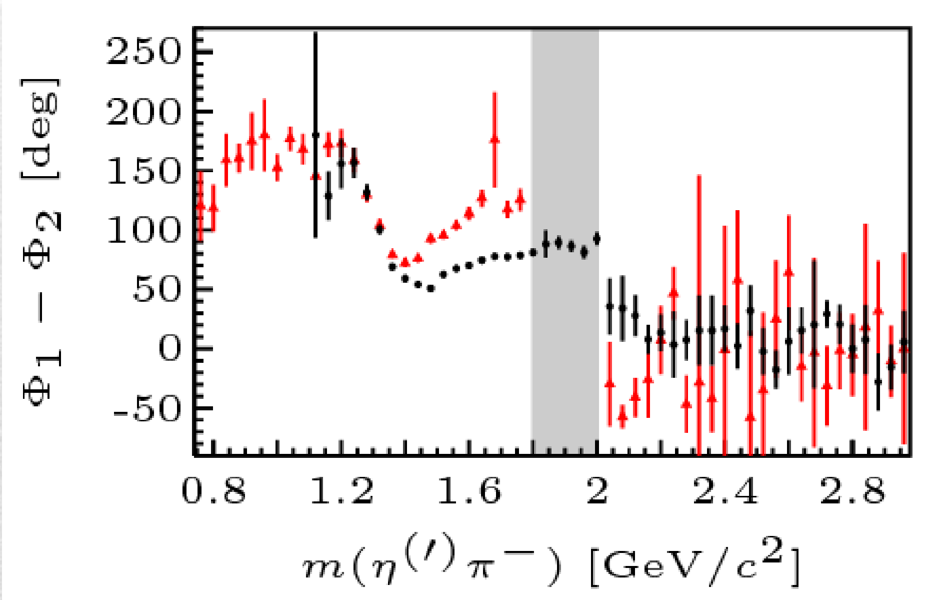
Precise determination of pole position

Data

COMPASS, PLB740, 303-311







A sharp drop appears at 2 GeV in *P*-wave intensity and phase

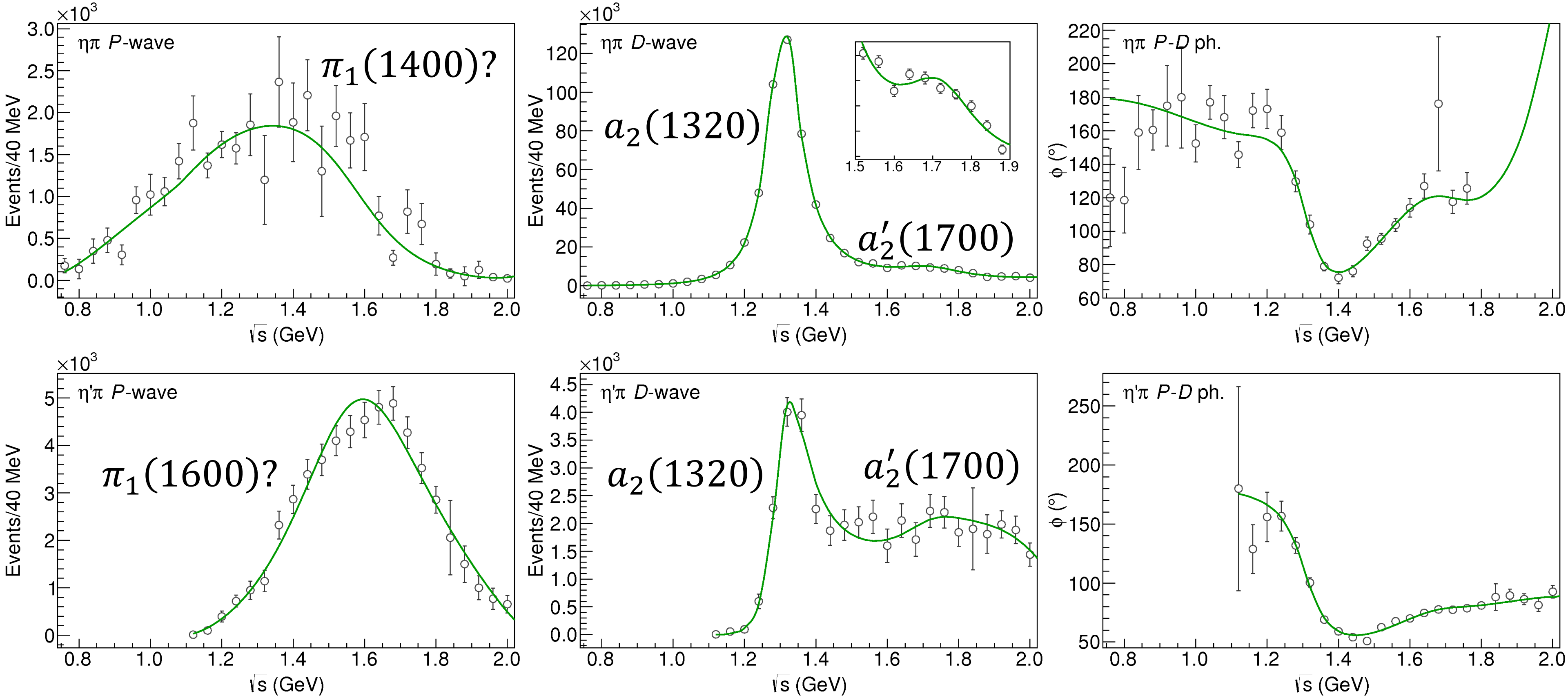
No convincing physical motivation for it

It affects the position of the $a_2'(1700)$

We decided to fit up to 2 GeV only

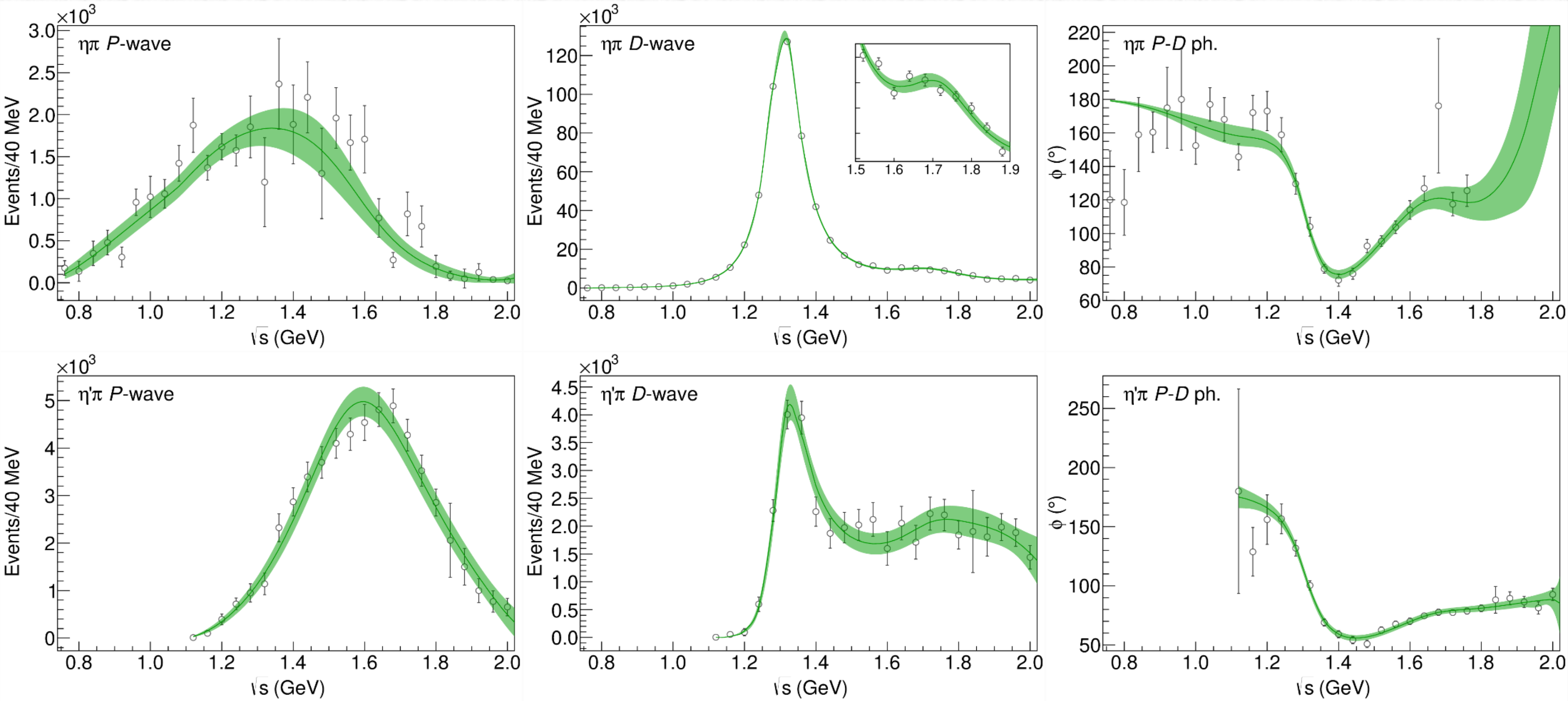
Fit to $\eta^{(\prime)}\pi$

(data)

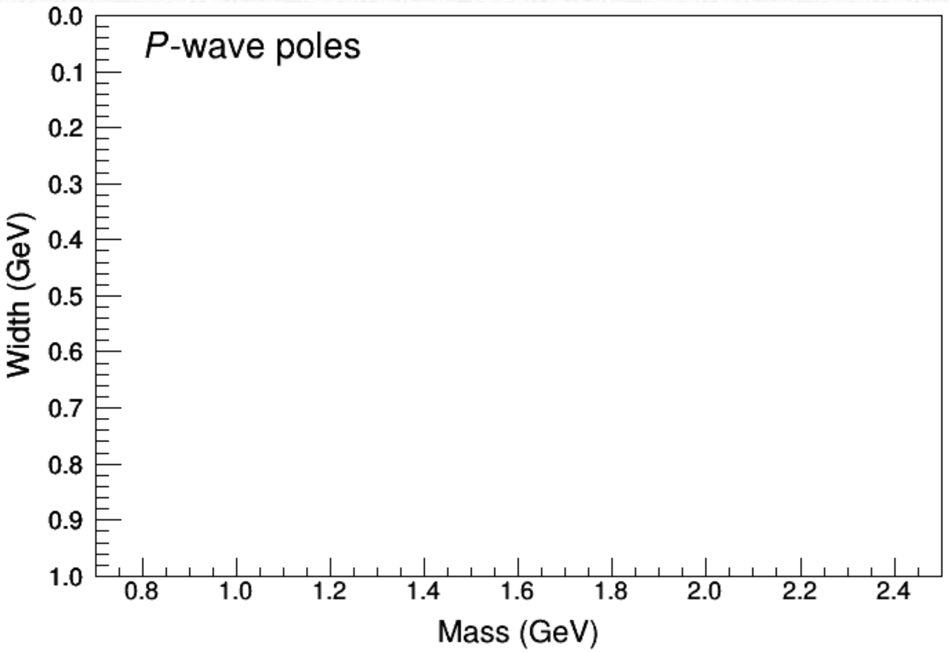


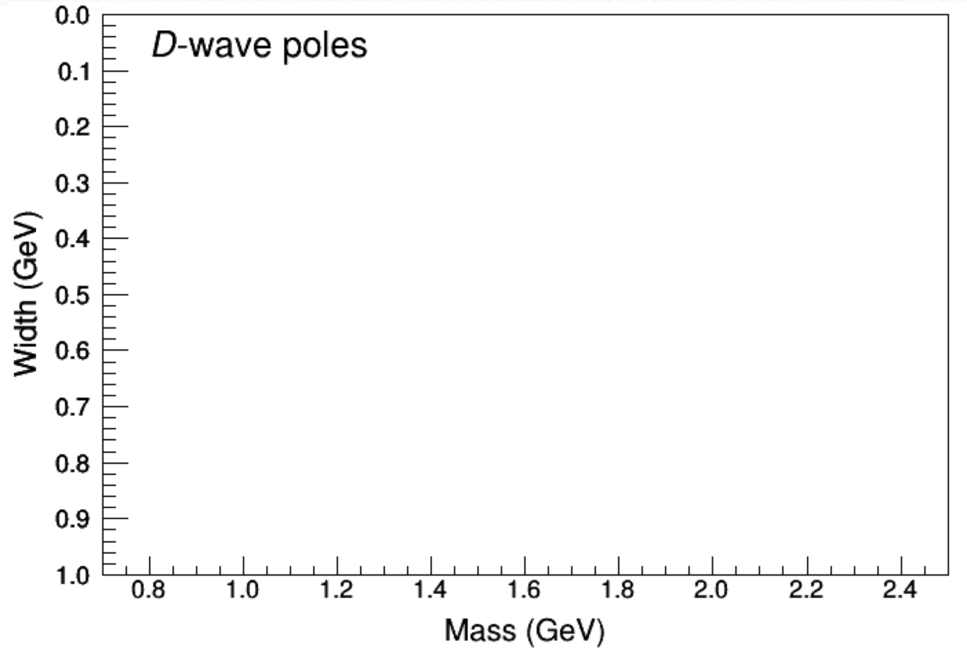
$J^{PC} = 1^{-+}$ $\qquad$ $J^{PC} = 2^{++}$

Statistical Bootstrap



Statistical Bootstrap





We can identify the poles in the region $m \in [1.2, 2]$ GeV, $\Gamma \in [0, 1]$ GeV

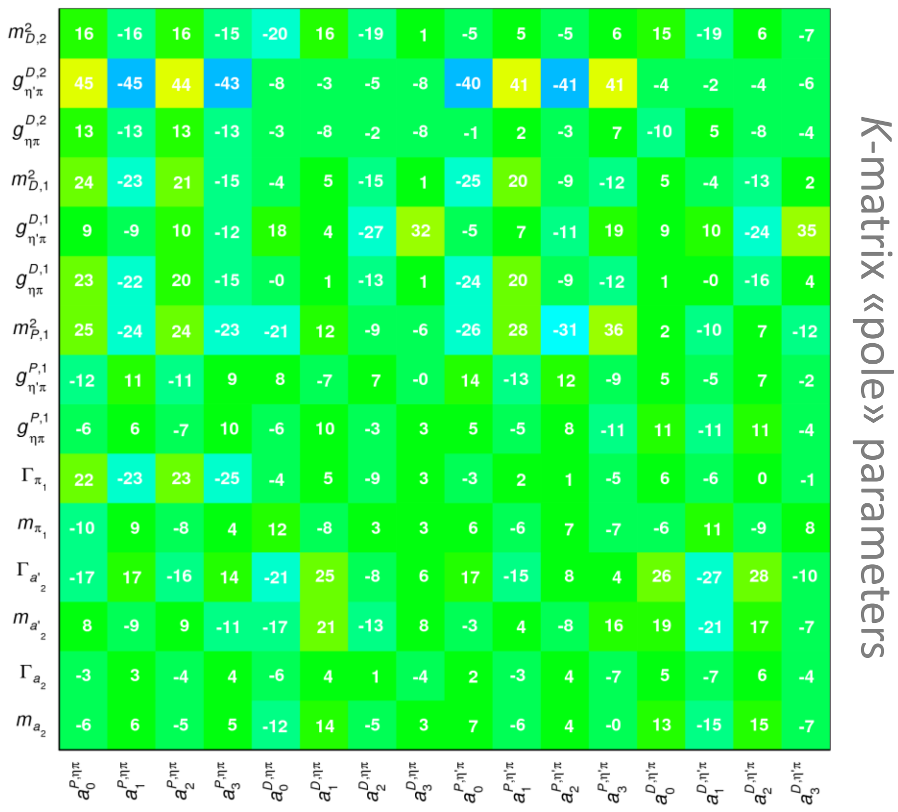
Two clusters in *D*-wave: $a_2(1320)$ and $a_2'(1700)$
Only one stable cluster in P-wave: a single π_1

Correlations

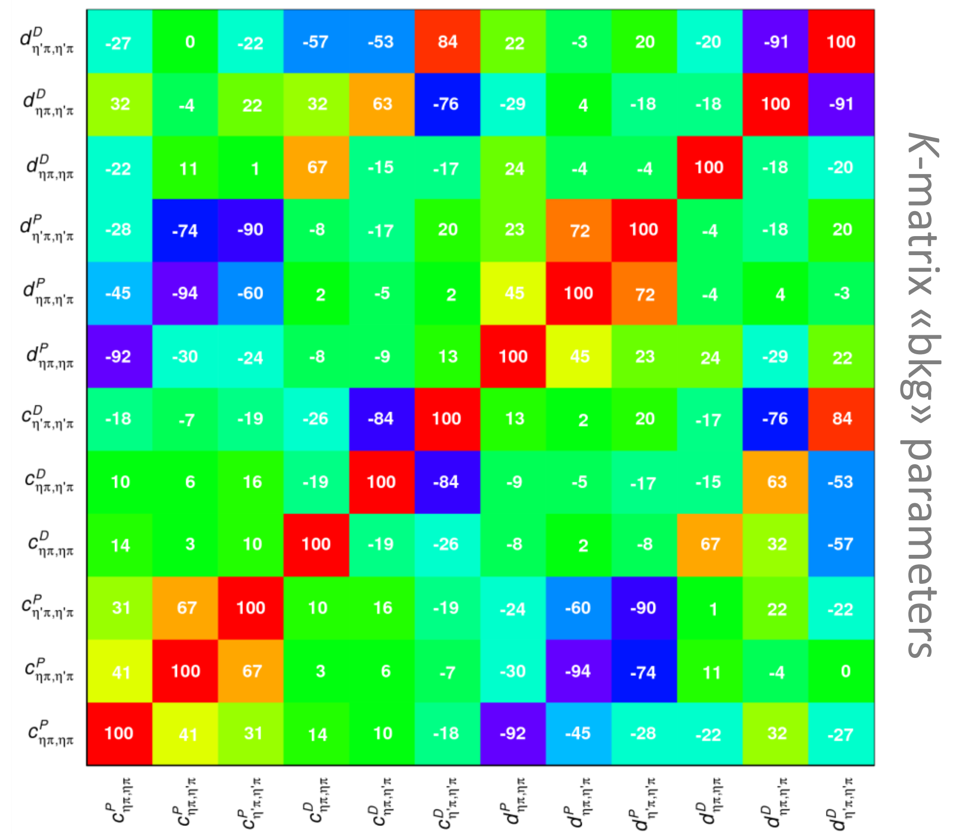
Denominator parameters uncorrelated with the numerator ones ✓

Denominator parameters uncorrelated between *P*- and *D*-wave ✓

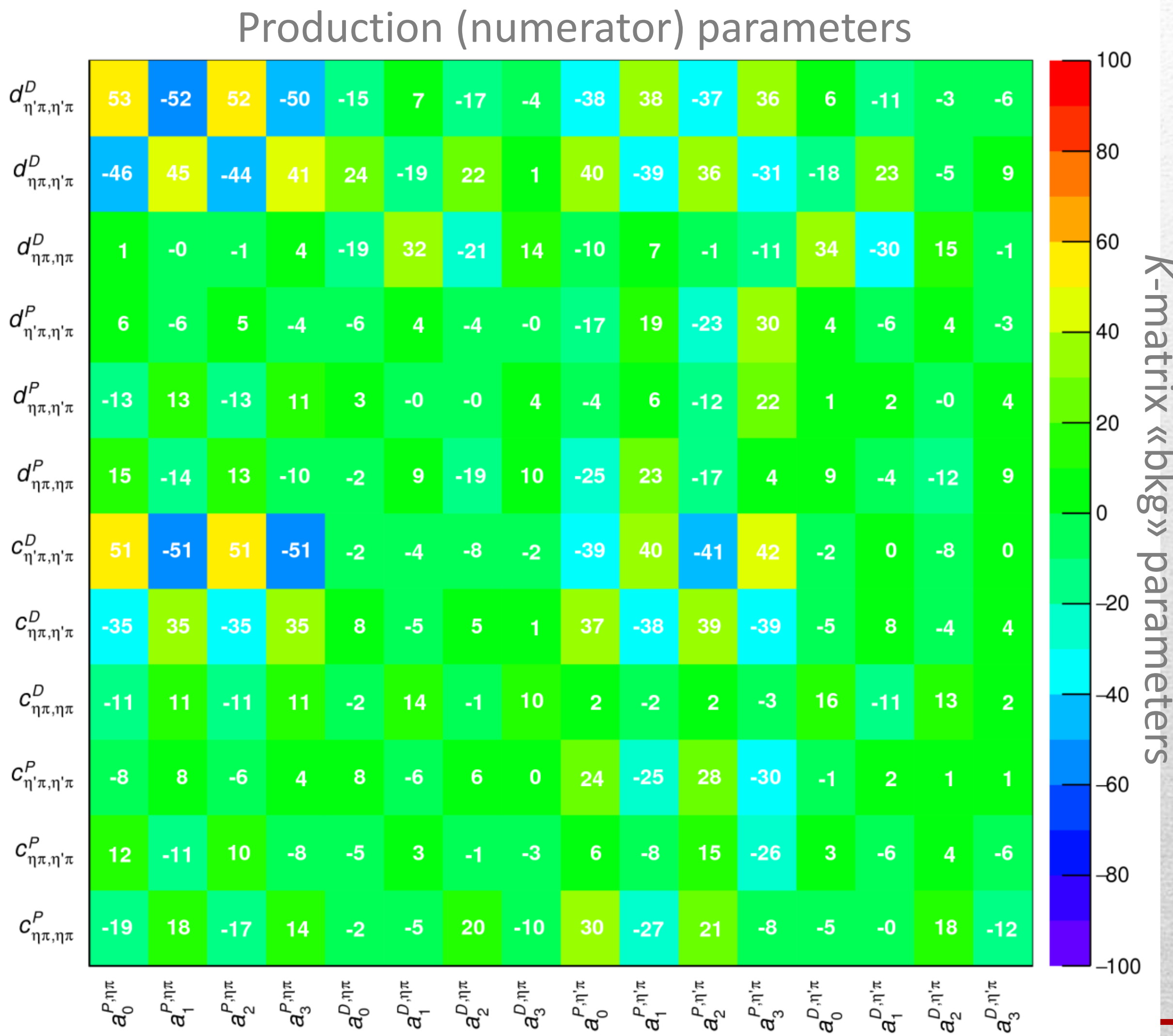
Production (numerator) parameters



K-matrix «bkg» parameters

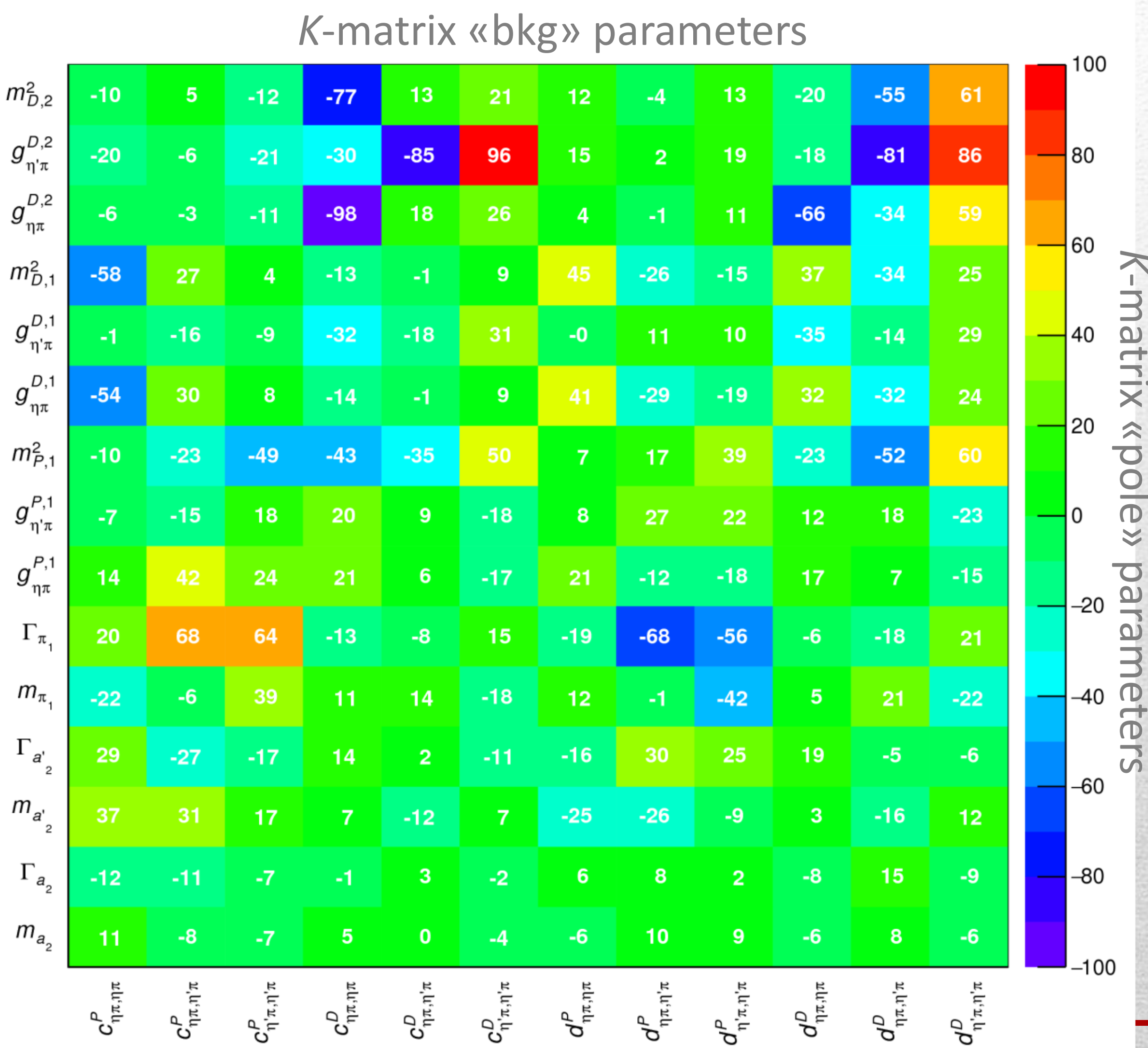


Correlations



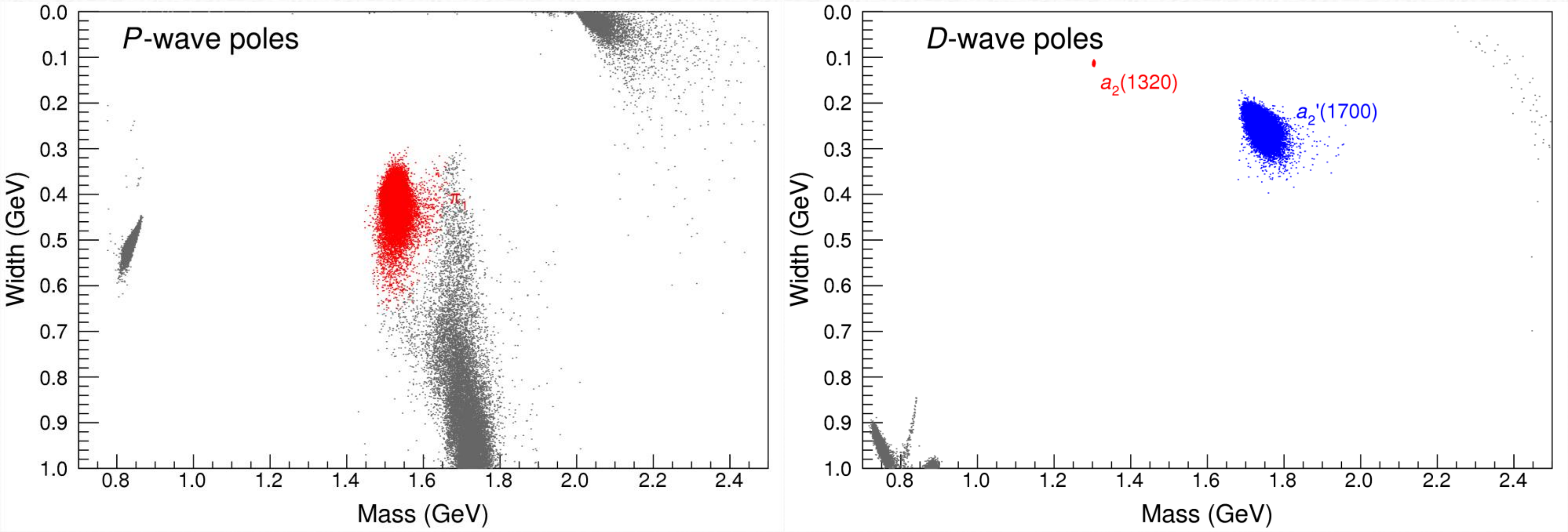
Denominator parameters not very correlated with the numerator ones ✓

Correlations



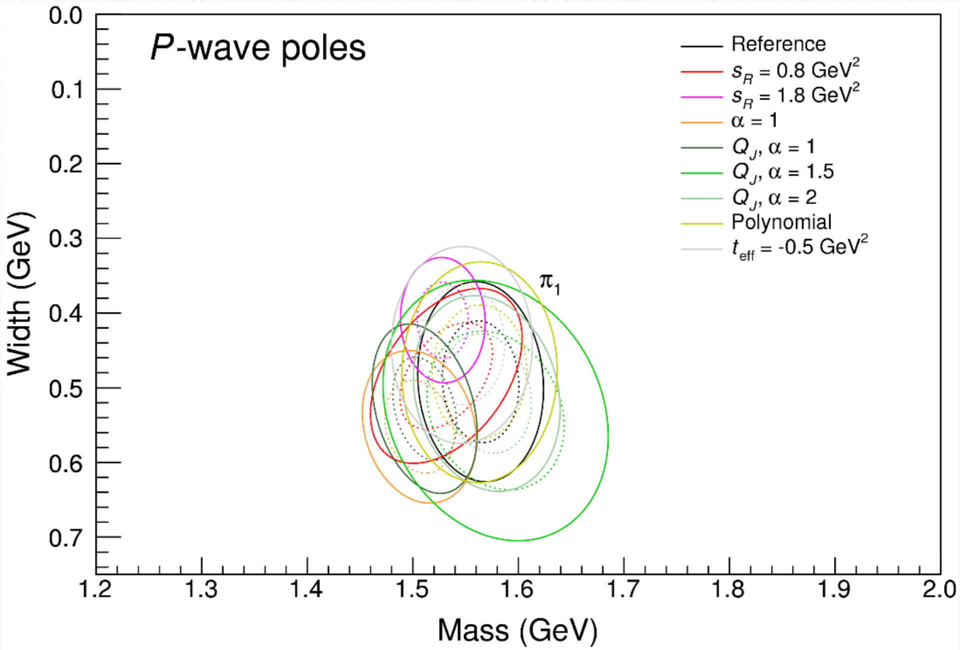
Denominator parameters uncorrelated between *P*- and *D*-wave ✓

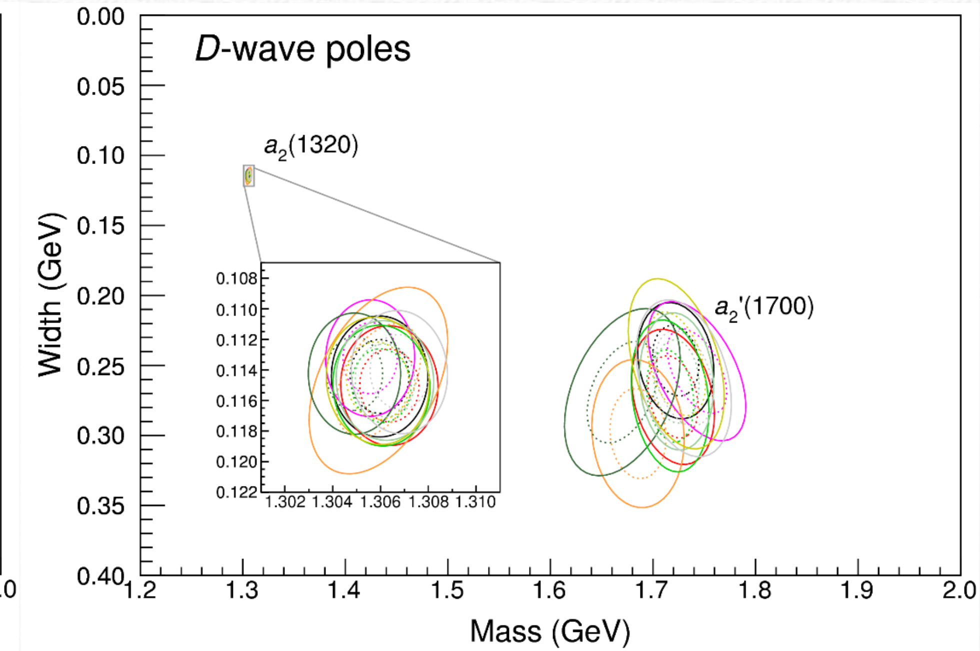
Bootstrap for $s_R = 1.8\ \mathrm{GeV}^2$



Our skepticism about a second pole in the relevant region is confirmed:
It is unstable and not trustable

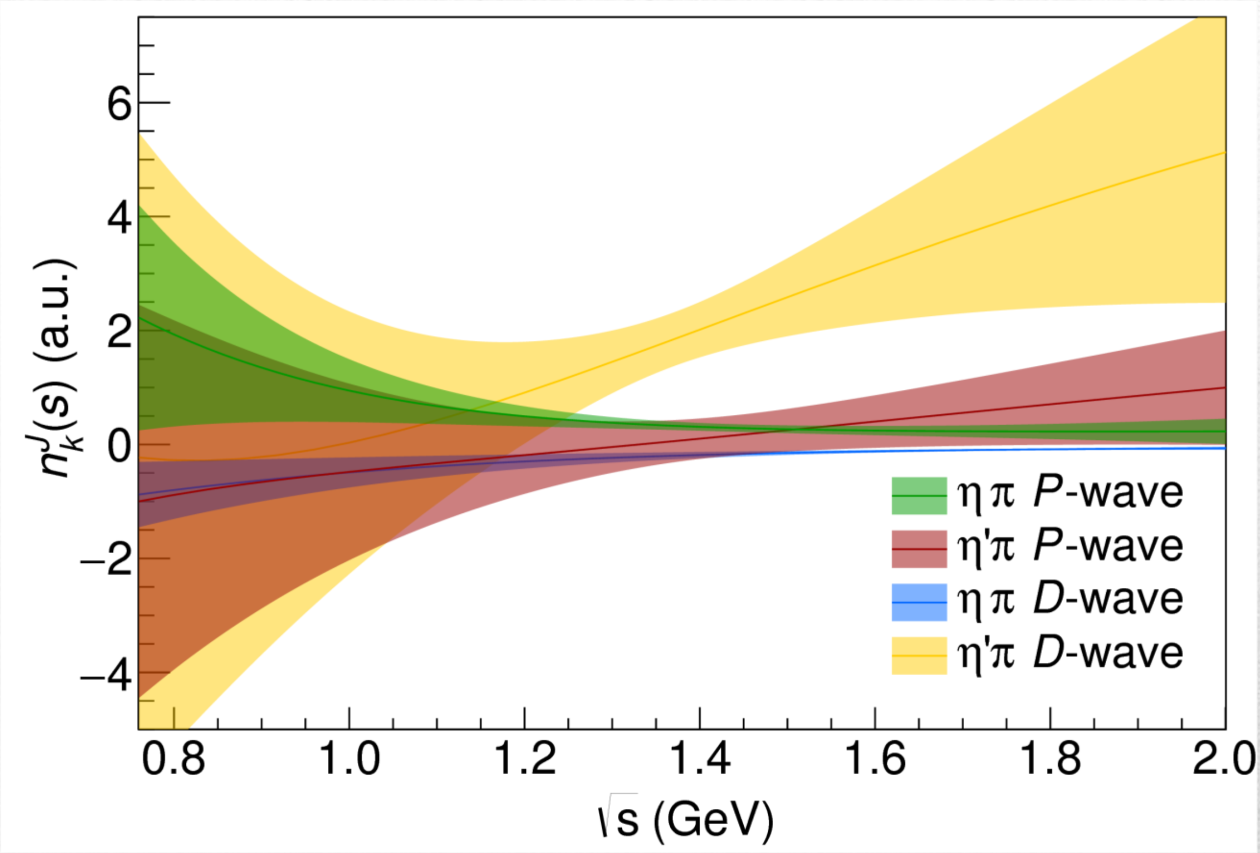
Systematic studies





For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

Polynomial in the numerator



The numerator should be smooth and have variation milder that the typical resonance width

This happens indeed

Coupled channel: the model

Two channels, $i, k = \eta\pi, \eta'\pi$ Two waves, $J = P, D$ 37 fit parameters

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s' - s - i\epsilon)}$$

$$K^J_{ki}(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c^J_{ki} + d^J_{ki}\, s$$

1 K-matrix pole for the P-wave
2 K-matrix poles for the D-wave

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{\lambda^{J+1/2}\left(s', m^2_{\eta^{(\prime)}}, m^2_\pi\right)}{(s' + s_R)^{2J+1+\alpha}}$$

$$n^J_k(s) = \sum_{n=0}^{3} a_n^{J,k}\, T_n\left(\frac{s}{s + s_0}\right)$$

Left-hand scale (Blatt-Weisskopf radius) $s_R = s_0 = 1\ \mathrm{GeV}^2$
$\alpha = 2$, 3rd order polynomial for $n^J_k(s)$

Systematic studies

- Change of functional form and parameters in the denominator

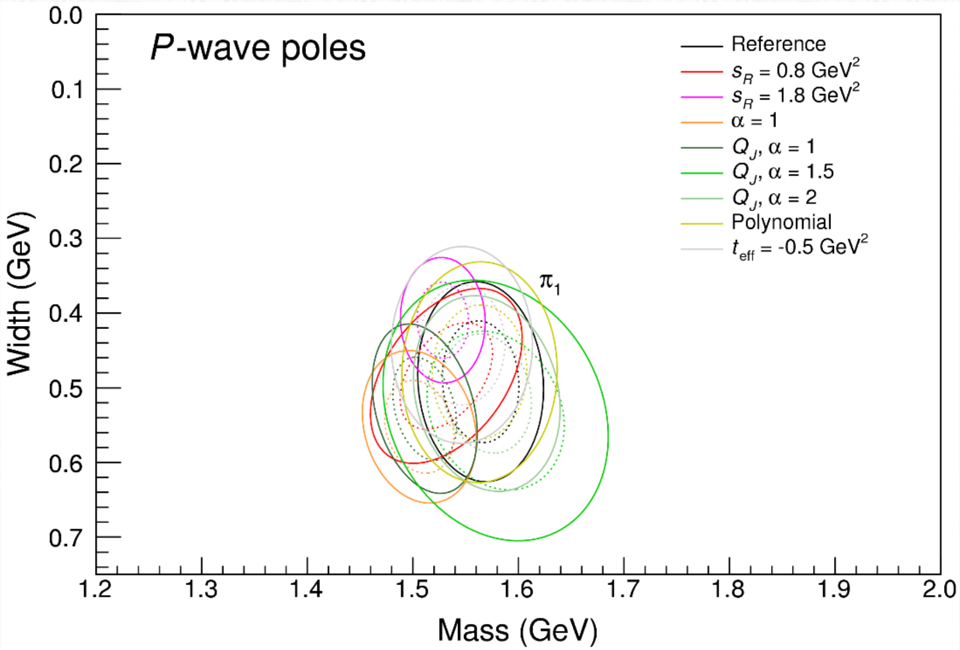
$$\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{\lambda^{J+1/2}\left(s', m_{\eta^{(\prime)}}^2, m_\pi^2\right)}{\left(s'+s_R\right)^{2J+1+\alpha}}$$

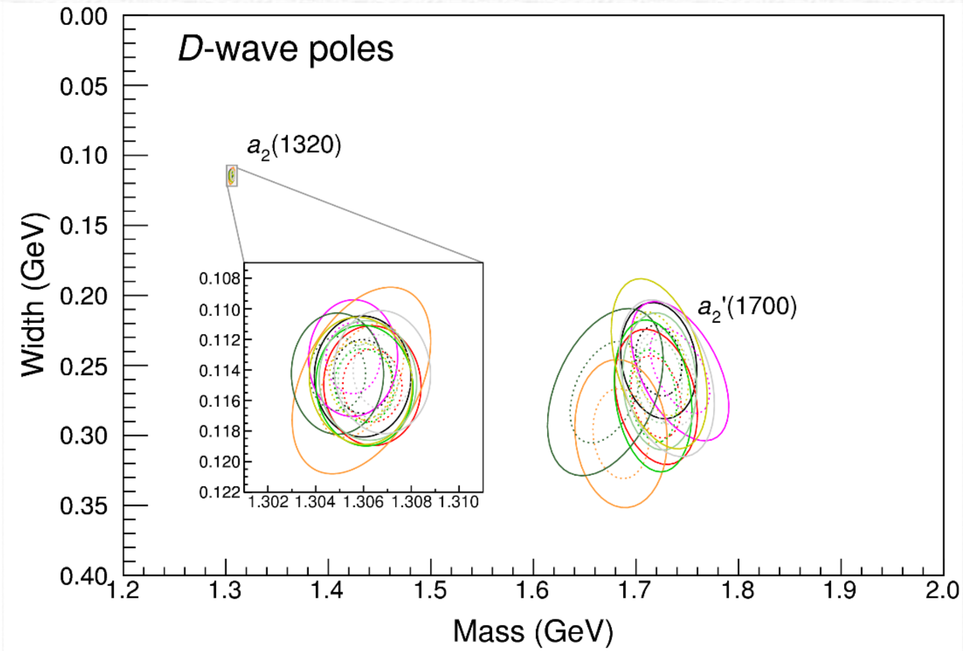
  - Default: $s_R = 1\ \mathrm{GeV}^2$. We try $s_R = 0.8, 1.8\ \mathrm{GeV}^2$
  - Default: $\alpha = 2$. We try $\alpha = 1$
  - We also try a different function: $\rho N_{ki}^{J}(s') = g\,\delta_{ki}\,\frac{Q_J(z_{s'})}{s'^{\alpha}\lambda^{1/2}(s', m_{\eta^{(\prime)}}, m_\pi)}$ with $\alpha = 2, 1.5, 1$

- Change of parameters in the numerator

  - Default: $t_{\mathrm{eff}} = -0.1\ \mathrm{GeV}^2$. We try$t_{\mathrm{eff}} = -0.5\ \mathrm{GeV}^2$
  - Default: 3rd order polynomial. We try 4th

Systematic studies





For each class, the maximum deviation of mass and width is taken as a systematic error
Deviation smaller than the statistical error are neglected
Systematic of different classes are summed in quadrature

Systematic studies

Systematic	Poles	Mass (MeV)	Deviation (MeV)	Width (MeV)	Deviation (MeV)
Variation of the function $\rho N(s')$					
$s_R = 0.8\,\mathrm{GeV}^2$	$a_2(1320)$	1306.4	0.4	115.0	0.6
	$a_2'(1700)$	1720	-3	272	26
	π_1	1532	-33	484	-8
$s_R = 1.8\,\mathrm{GeV}^2$	$a_2(1320)$	1305.6	-0.4	113.2	-1.2
	$a_2'(1700)$	1743	21	254	7
	π_1	1528	-36	410	-82
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		21		26
	π_1		36		82
$\alpha = 1$	$a_2(1320)$	1305.9	-0.1	114.7	0.3
	$a_2'(1700)$	1685	-37	299	52
	π_1	1506	-58	552	60
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		37		52
	π_1		58		60
$Q_J, \alpha = 1$	$a_2(1320)$	1304.9	-1.1	114.2	-0.2
	$a_2'(1700)$	1670	-52	269	22
	π_1	1511	-53	528	36
$Q_J, \alpha = 1.5$	$a_2(1320)$	1306.0	0.1	115.0	0.6
	$a_2'(1700)$	1717	-5	272	25
	π_1	1578	14	530	39
$Q_J, \alpha = 2$	$a_2(1320)$	1306.2	0.2	114.7	0.3
	$a_2'(1700)$	1723	1	261	15
	π_1	1570	6	508	16
Systematic assigned	$a_2(1320)$		1.1		0.0
	$a_2'(1700)$		52		25
	π_1		53		0

Systematic studies

Variation of the numerator function $n(s)$

Polynomial expansion	$a_2(1320)$	1305.9	-0.1	114.7	0.3
	$a_2'(1700)$	1723	1	249	2
	π_1	1563	-1	479	-13
Systematic assigned	$a_2(1320)$		0.0		0.0
	$a_2'(1700)$		0		0
	π_1		0		0
$t_{\text{eff}} = -0.5\,\text{GeV}^2$	$a_2(1320)$	1306.8	0.8	114.1	-0.3
	$a_2'(1700)$	1730	8	259	13
	π_1	1546	-18	443	-49
Systematic assigned	$a_2(1320)$		0.8		0.0
	$a_2'(1700)$		0		0
	π_1		0		0

Double Regge Exchange Model

Shimada *et al.*, NPB

$$T^{\tau_1\tau_2} = -K\tilde{T}^{\tau_1\tau_2} = -K\Gamma(1-\alpha_1)\Gamma(1-\alpha_2)$$
$$\left[(\alpha' s)^{\alpha_1-1}(\alpha' s_2)^{\alpha_2-\alpha_1}\xi_1\xi_{21}\hat{V}_1 + (\alpha' s)^{\alpha_2-1}(\alpha' s_1)^{\alpha_1-\alpha_2}\xi_2\xi_{12}\hat{V}_2\right]$$

where

$$\hat{V}_1(\eta, t_1, t_2) = \beta_0 \frac{\Gamma(\alpha_1-\alpha_2)}{\Gamma(1-\alpha_2)}\, {}_1F_1\left(1-\alpha_1, 1-\alpha_1+\alpha_2, -\frac{1}{\eta}\right)$$

and $\hat{V}_2$ is obtained by replacing $\alpha_1 \leftrightarrow \alpha_2$.

Double Regge Exchange Model

Shimada *et al.*, NPB

- Signature factors are defined as:

$$\xi_i = \frac{1}{2}(\tau_i + e^{-i\pi\alpha_i}) \qquad \xi_{ij} = \frac{1}{2}(\tau_i\tau_j + e^{-i\pi(\alpha_i-\alpha_j)})$$

  and kinematic singularities factored:
  $K = -4\sqrt{s_1}|\mathbf{p}_a||\mathbf{p}_1||\mathbf{p}_3|\sin\theta_2\sin\theta_{GJ}\sin\phi_{GJ}$
- Both α_1 and α_2 are of 2^{++} type so we put $\tau_1 = \tau_2 = +1$.
- Regge trajectories:

$$\alpha_{f_2}(t) = \alpha_{a_2}(t) = 0.47 + 0.89t \qquad \alpha_{\boldsymbol{P}}(t) = 1.08 + 0.25t$$

Fit to $\eta\pi$ in bins of energy